Classifying Polynomials

A **polynomial** is an algebraic expression with one or more *unlike* terms linked together by + or –

**Polynomials can be classified by the number of terms they have:
A ***monomial*** has ***1 term***, a **binomial** has **2 terms**, and a **trinomial** has **three terms**.

**Polynomials can be classified by degree:
The **degree of a term** is the sum of the exponents on the variables in the term
The **degree of the polynomial** is the degree of the highest-degree term

Polynomial	# of Terms	Degree of Each Term	Degree of the Polynomial
$x + 3$			
$5x^2 - 2x$			
$3y^3 + 0.2y - 1$			
$7x^2y^4 + x^6y$			
$0.7u - 2a^2b$			
$abxy - 8ab^2 + xy$			

Simplifying Polynomials

Polynomials can be SIMPLIFIED by using exponent laws and collecting like terms. Once a polynomial is simplified, nothing more can be done without more information about your variable.

Remember: both LIKE and UNLIKE terms can be multiplied and divided, but only LIKE terms can be added or subtracted.

Simplifying by adding and subtracting like terms:

1. Add/subtract the coefficient (number in front)
2. Keep the variable (letter) and exponent the same.

Simplify the following expressions:

$3x + 4x$ $10y - 4y$

When there are more than a couple of terms, first COLLECT the like terms together, then SIMPLIFY:

$4x + 8 - 2x + 4$

Collect like terms:

Simplify:

$$9 + 3x^2 + 4x + 2x^2 - 5 - 6x$$

Collect like terms:

Simplify:

$$6xy^2 + 5y - 6 + 3y^2x - 7y + 6$$

Collect like terms:

Simplify:

Practice: Simplifying

1. Which polynomial contains a term like xy^2?

A $4xy - x^2y$ **B** $2x^2 + 3xy^2$ **C** $-x + y^2 - xy$ **D** $x^2 + y^2 + 4$

2. Are the terms in each pair like or unlike?

a) $5a$ and $-2a$ **b)** $3x^2$ and x^3 **c)** $2p^3$ and $-p^3$

d) $4ab$ and $\frac{2}{3}ab$ **e)** $-3b^4$ and $-4b^3$ **f)** $6a^2b$ and $3a^2b$

g) $9pq^3$ and $-p^3q$ **h)** $2x^2y$ and $3x^2y^2$

3. Write one like term and one unlike term for each.

Term	Like	Unlike	Term	Like	Unlike	Term	Like	Unlike
$4p$			$-3a^2$			$-k^3$		
$2x$			$-4mn^4$			$2ab$		
$-pq^3$			$3b^2d^2$			$-7b^5$		

4. Is it possible to simplify each expression? How do you know?

a) $8a + 3a$ **b)** $5m + 2n$ **c)** $3p + p$

d) $3t - 7t$ **e)** $4x - 3$ **f)** $-v - 4v + 2v$

g) $6c^2 - c^2 - 3c^2$ **h)** $r^2 + 3r + 7$

5. Simplify each expression.

a) $p + 2p$ **b)** $7g - 4g$ **c)** $2a - 8a$

d) $5x - 2x$ **e)** $6q + q$ **f)** $4y^2 + 5y^2$

g) $u + 4u - u$ **h)** $7b^3 - 2b^3 - b^3$

6. Collect like terms. Then, simplify.

a) $4b + 3 - 2b + 1$ **b)** $2p - 7 - p + 4$ **c)** $1 + 3y + 4 + y$

d) $5 - x - 1 - 2x$ **e)** $6a - 2b + 3b + 2a$ **f)** $7r + 2 + 3r - r - 1$

g) $9s - 2s + 5t - 4s$ **h)** $-g - 3h + 5h + 2g - h$

7. Simplify.

a) $4 + v + 5v - 10$ **b)** $7a - 2b - a - 3b$ **c)** $8k + 1 + 3k - 5k + 4 + k$

d) $2x^2 - 4x + 8x^2 + 5x$ **e)** $12 - 4m^2 - 8 - m^2 + 2m^2$ **f)** $-6y + 4y + 10 - 2y - 6 - y$

g) $5 + 3h + h - 4 + h + 6 + 2h$ **h)** $4p^2 + 2q^2 - p^2 + 3p^2 - 7q^2$

8. Simplify.

a) $2a + 6b - 2 + b - 4 + a$ **b)** $4x + 3xy + y + 5x - 2xy - 3y$

c) $m^4 - m^2 + 1 + 3 - 2m^2 + m^4$ **d)** $x^2 + 3xy + 2y^2 - x^2 + 2xy - y^2$

Building Polynomial Expressions

An **expression** contains numbers and variables. An **equation** also contains numbers and variables, but it also contains an equal sign. An equation says that two expressions are equal.

For example: $3x + 5$ is an expression. $4x - 2$ is another expression.

$3x + 5 = 4x - 2$ is an equation that says, "$3x + 5$ is equal to $4x - 2$"

Defining Variables: When we choose variables to use in expressions, we first have to DEFINE them (say what they're representing).

1. Ms. Bello works part-time as a scuba instructor. She earns $145 for the summer, plus $15 for each children's lesson and $30 for each adult lesson that she gives.

a) Write an expression that describes Ms. Bello's total earnings for the season. ***Define all variables***. Identify the variable and the coefficient of each term and explain what they mean.

Term	Variable	Meaning of Coefficient

b) One summer, Ms. Bello gave 9 children's lessons and 14 adult lessons. What were her total earnings?

2. The students at Northdale High School sell coupon books to raise money for a school trip. The school receives 45% of the money paid for the coupon books.

a) Choose a variable to represent the money paid for the coupon books. Define it.

b) Using your variable from part a), write the expression for the amount of money the school will receive.

c) Shannon sold one coupon book to her grandmother for $20. Calculate the amount of money the school receives on this sale.

d) The sum of all coupon book orders was $14 000. Use your formula to calculate how much the school will receive for this fundraiser.

The Distributive Property

$5(4 + 3)$ ← There is more than one way to solve this problem.

Solution #1:

Solution #2:

Solution #2 is called the DISTRIBUTIVE PROPERTY. When you use this property, you are expanding.

Expand and simplify the following (if possible):

a) $4(x + 3) + x - 5$

b) $-(2x + 7)$

c) $x(x^2 + 2x - 4)$

d) $-4(x + 3) - 2(2x - 1)$

e) $4m(m - 2) - (2m^2 - m)$

f) $\frac{1}{2}(2w - 6) - \frac{2}{3}(6w - 3)$

PART 1 Practice: Expand and simplify (if possible).

1. $2(x - 4)$

2. $p(p^2 - 2p + 1)$

3. $-5(4m - 3)$

4. $-3h(4-h^2)$

5. $-(-w+5)$

6. $(x^3+3x-4)(4x)$

7. $(2y+5)(-6)$

8. $-4(d+3)+(d-1)$

9. $3(5a^2-7a+1)$

10. $3m(m-5)-(2m^2-m)$

11. $(x-3)+(2x-5)$

12. $3\left[2+5(2k-1)\right]$

13. $5\left[4a-(a+2)\right]$

14. $2\left[3c-(c-2)\right]-3\left[2c+(c+3)\right]$

15. $-y(2y-7)$

16. $3\left[-2(6-t)+5t\right]$

17. $3(a+2)+5(a-3)-(a+4)$

18. $3x(2x+3)+4(x^2+2x-4)$

19. $5g(2g-3)-3(2g^2-4g+3)$

20. $\frac{1}{2}(2w-6)-\frac{2}{3}(9w-6)$

21. $\frac{2}{3}(3m+5)+\frac{2}{5}(5m-4)$

Word Problems:

1. A room has dimensions as shown:

a) Find a simplified expression for the perimeter.

b) Find a simplified expression for the area.

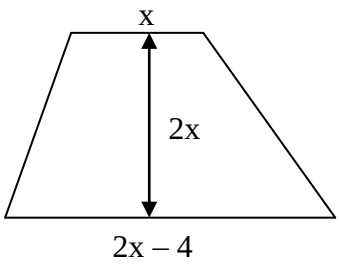
c) Repeat parts a) and b) if both the length and width are doubled.

d) Has this doubled the perimeter? Justify your solution.

e) Has this doubled the area? Justify your solution.

2. A rectangle has side lengths of (2x+5)cm and (5x–9) cm. Determine the simplified expression for the **perimeter** of the rectangle.

3. Steven is an architect designing a front entranceway in the shape of a large trapezoid (displayed below). To order tiling materials, he needs to determine the trapezoid's area. Write a simplified expression for the area of this trapezoid:



4. Laura claims, "I can calculate the perimeter of the field by using the formula P = 2 (l + w)"
Tyler replies, "That's not right. The correct formula is P = 2l + 2w" Who is correct, Laura or Tyler? Can they both be right?

More Practice

5. Expand.

a) $4(x + 2)$ **b)** $5(x - 3)$ **c)** $0.3(x + 5)$ **d)** $4(2x + 1)$

e) $\frac{1}{2}(3x - 2)$ **f)** $5(3 + 2x)$ **g)** $a(a + 3)$ **h)** $s(s - 5)$

i) $-y(y + 2)$ **j)** $b(4 - b)$ **k)** $-x(6 - x)$ **l)** $-k(k - 3)$

m) $4r(r + 3)$ **n)** $6m(m - 2)$ **o)** $2x(3 - x)$ **p)** $-3y(5 + y)$

6. Expand and simplify.

a) $3x + 2(5x - 3)$

b) $14 - 3\left(4n - \frac{1}{3}\right)$

c) $3(2h - 3) + 2(h + 3)$

d) $-2(3y - 3) + 3(2y + 2)$

e) $-6 + 5(2 - k) - 4k$

f) $4(3u - 1) + 2(3 - 2u)$

g) $2(x^2 + 2x + 1) + 3(x^2 + 3)$

h) $5(y - 2) - 4\left(2y - \frac{1}{2}\right)$

i) $3(t^2 - 2t + 1) - 4(t + 2)$

j) $2(e - 4) + 4(3e + 2) - 5(2e - 4)$

k) $x(2x - 3) - x(4 + x)$

l) $2a(a + 2) + 4a(a + 1)$

m) $3r(r - 3) - 2r(r + 2)$

n) $k(4k - 2) - k(k + 3)$

o) $-d(3 - d) + 2d(d + 5)$

p) $4x(x - 1) - x(2 - x)$

q) $2(a^2 + 3a - 10) - a(a + 2)$

r) $3x(x^2 + 2x - 8) - 2(x - 1)$

s) $2(y - 1) + y(y^2 - y - 2)$

t) $-2r(r + 5) + 3r(r - 3)$

Common Factoring

Factoring is the opposite of the distributive property. The greatest common factor (**GCF**) for a polynomial is the largest monomial that you can divide out of each term in the polynomial.

Steps:	**Ex 1:** $6x^2 - 8x$	**Ex 2:** $9x^2y^2 + 6xy^2 - 12x^3y^3$
STEP 1: Look at the coefficients. Is there a GCF?		
STEP 2: Look at the variables. Is there a variable that is common in every term? If so, take out the smallest exponent.		
STEP 3: Identify the GCF. Then, divide every term by the GCF (this is the left over that will go into the brackets).		
STEP 4: Write it appropriately in factored form.		
***TO CHECK:** You multiply the GCF back into every term to see if it matches the original polynomial.		

Examples: Factor fully.

1. $6x + 3$
2. $49p - 14$
3. $3x^2 + 9x - 3$
4. $16x^3 + 8x^2 + 4x$
5. $20x^3y^2 + 5x^4y - 10x^2y^2$
6. $9x^3+6x^5+12x^2$
7. $8xy^2 + 4x^2y - 6xy^5$
8. $3x - 6$
9. $15x + 10y + 25$

10. $4x - 4y + 8$

11. $12x^2 - 6x + 9$

12. $x^8 + x^7 + x^6 + x^5$

13. $5x^5 - 4x^4 + 3x^3$

14. $x^3 + x^2$

15. $6x^5 + 2x^3$

16. $2x^3 - 4x^2 + x$

17. $3x^6 - 2x^5 + 4x^4 - 6x^2$

18. $16x^5 - 32x^4 + 24x^3$

19. $36y^{15} - 27y^{10} - 18y^5$

20. $8z^2 - 12z + 20$

21. $16x^2 - 24x + 40$

22. $20x^4 - 12x^3 + 36x^2 - 4x$

23. $18x^8 - 81x^6 + 27x^4 - 45x^2$

24. $12x^{10} - 6x^3 + 3$

25. $3abc - 4ab$

26. $2xy - 8xyz$

27. $x^2y^3 - x^3y^2$

28. $8ab^3 + 12a^2b^2$

29. $a^5b^5 - a^8b^2$

30. $x^6yz^2 + x^2y^4z^3 - x^3y^3z^4$