## Solving One-step Equations (Addition and Subtraction)

Equation: has an equal sign
Expression: does NOT have an equal sign
Ex 1: I have a bag with an unknown number of marbles in it.
The marbles in my bag, plus 5 more marbles, is the same as 12 marbles. How many marbles do I have?

Define your variables: Let $\qquad$ represent $\qquad$

## Using pictures:

## Using opposite operations:

## Final statement:

When solving equations, the goal is to isolate the variable (which means get your variable by itself on one side of the equal sign)

$$
\text { Ex 2: Solve } \quad x+13=75
$$

Ex 3: Solve $\quad \mathbf{y - 2 8}=\mathbf{9}$

Practice: Solve the following.
$x+9=28$

$$
43+g=324
$$

$d-20=42$
$f+34=9$

## Solving One-step Equations (Multiplication and Division)

Ex 4: I have a bag with an unknown number of marbles in it.
3 of my bags is the same as 12 marbles. How many marbles are in my bag?

Define your variables: Let $\qquad$ represent $\qquad$

## Using pictures:

## Using opposite operations:

Final statement:

$$
\text { Ex 5: Solve } \quad 13 x=156
$$

Ex 6: Solve $\quad \frac{x}{5}=17$

Practice: Solve the following by using opposite operations:
$9 x=131$
$-6 \mathrm{~g}=89$
$-d=46$

$$
\frac{x}{-4}=-57
$$

## Solving One-step Equations - Mixed Practice

Solve the following:
a) $q-1=8$
b) $\frac{m}{-3}=11$
c) $n+9=15$
d) $v-2=4$
e) $j+7=18$
f) $y-6=8$
g) $-4 x=36$
h) $d-5=4$
i) $-3 k=18$
j) $b-3=12$
k) $-c=3$
I) $-\frac{x}{4}=6$
m) $k+24=15$
n) $\frac{\mathrm{j}}{-14}=6$
о) $p+12.5=9$
p) $y-\frac{1}{2}=8$
q) $d+17.3=18.9$
r) $a-7=6$
s) $-7 x=30$
t) $d-1=\frac{2}{3}$
u) $-9 k=15$
v) $b-8=3.9$
w) $-c=18.26$
x) $-\frac{\mathrm{x}}{9}=1.6$

## Solving Two-step Equations

Do the opposite operations for addition and subtraction BEFORE multiplication and division.

Ex 7: Solve $3 x+4=10$
Ex 8: Solve $\frac{y}{5}-8=31$

## Check:

## Check:

**To CHECK your answer, substitute your solution back into the equation, and evaluate EACH side of the equal sign. If the left side equals the right side ( $\mathbf{L S}=\mathbf{R S}$ ), then your solution is correct**

Ex 9: Solve $\mathbf{6 j - 7}=\mathbf{1 0}$
Ex 10: Solve $\frac{\boldsymbol{k}}{\mathbf{4}}+7=15$

## Check:

Check:

## Solving Two-step Equations - Practice

Solve and check each of the following:
a) $5 k-3=82$
Check:
b) $-b+8=33$
Check:
c) $2 g-1=50$
Check:
d) $-3 s+2=-13$
Check:
e) $4 r+5=2$
Check:
f) $-3 k=14$
Check:
g) $\frac{\mathrm{b}}{3}-3=19$

Check:
h) $6 w-4=-22$

Check:
i) $-2 g+3=-4$

Check:
j) $5 s+3=2$

Check:
k) $3 d-5=-1$

Check:

1) $\frac{5 x}{4}-9=21$

Check:

## Solving Multi-step Equations

ALWAYS simplify first (and use the distributive property if needed)

Ex 11: Solve $3+\mathbf{4 m}+\mathbf{5 m}=\mathbf{2 1}$
Ex 12: Solve 6p-5=8p-9

## Check:

Ex 13: Solve $4(2 k-3)=4$
Ex 14: Solve $\mathbf{3}(\mathrm{k}-\mathbf{2})=\mathbf{2}(\mathrm{k}+8)$

Check:
Check:

Solve for the unknown variable.
1.) $-76=6 h+2 h+4$
2.) $2(-3 f-7)=-80$
3.) $2(5 s-7)=66$
4.) $40=3 \mathrm{k}+6 \mathrm{k}+4$
5.) $38=4 w+w+8$
6.) $-7+4 p+5 p=83$
7.) $-3(3 a+1)=78$
8.) $5(6 d-10)=-50$
9.) $71=3 n+7 n+1$
10.) $7 c+8+4 c=8$
11.) $6 b-5=-77-2 b$
12.) $4 m-9=3 m-5$
13.) $-1+3 z=z-9$
14.) $x-9=3+7 x$
15.) $-3 r+4=44+2 r$
16.) $-7(1 y-2)=0$
19.) $-5 p-3=-43+3 p$
22.) $-2 d-9+5 d=18$
23.) $5(10-\mathrm{f})=40$
26.) $-21=-4 h+6 h+3$
25.) $-2(10+6 h)=-116$
28.) $10+4 j+5 j=-98$
29.) $-6(10-3 \mathrm{k})=30$
18.) $-5 i+7=-2 i+7$
17.) $3 u+1=111-7 u$
21.) $6(-8-2 s)=36$
20.) $2 \mathrm{a}-1=-2 \mathrm{a}+43$
24.) $-7+4 g=-2 g+29$
27.) $4(-3 h+4)=-80$
30.) $3+3 z+2 z=28$

## WRITMG EXPRZSSIONS AND EQUATIONS

There are certain phrases that can help you figure out what you need to do in the word problem (listed in the chart below). HOWEVER, you still need to read the question carefully (don't just hunt for keywords) because certain words can mean different things in different situations.

| Add | Subtract | Multiply | Divide | Equal to |
| :--- | :--- | :--- | :--- | :--- |
| plus/ sum | decreased by | of | per, a | is, are, was, were, will |
| increased by | minus, less | times, multiplied by | out of | be |
| more than | difference between/of | product | ratio of, quotient of | gives, yields |
| combined, together | less than, fewer than | double/triple/etc | divided by | sold for |
| total of | loss | twice/three times/etc | divided/shared |  |
| added to | take away | percent of | equally/equal pieces |  |
| gain/raise | reduce | by | cut/split |  |
| and | fell/dropped | by a factor of | each/every |  |
| in all/altogether |  |  | average |  |
| additional/extra |  |  |  |  |

## Addition phrases

- the sum of a number and four
- four more than a number
- a number increased by four


## Subtraction phrases

- the difference between $y$ and five
- five less than a number
- a number decreased by five
- five subtracted from a number


## Multiplication phrases

- the product of seven and $x$
- seven multiplied by $x$
- five percent of $x$
- $5 \%$ of $x$


## Consecutive Integer phrases

- sum of three consecutive integers
- sum of three consecutive even integers
- sum of three consecutive odd integers
- sum of squares of three consecutive even integers
- sum of squares of three consecutive odd integers
- product of two consecutive even integers
- product of two consecutive odd integers

Fractional phrases

- three-fourths of a number
- one-third the sum of a number and two
- half the result of decreasing a number by three
- a fraction whose denominator is two more than its numerator


## Practice Writing Expressions and Equations

Translate the following into algebraic EXPreSSIOnS, then simplify if possible. DO NOT solve. Let "n" represent the number:

1. the sum of 8 and a number: $\quad n+8$
2. 4 less than a number: $\qquad$
3. my number multiplied by 13 : $\qquad$
4. the quotient of my number and 3 : $\qquad$
5. six less than four times the sum of two and a number: $\qquad$
6. twice the square of a number: $\qquad$
7. the difference of 5 and my number: $\qquad$
8. nine less than the total of a number and two: $\qquad$
9. seven less than three times a number: $\qquad$
10. the square of the sum of $x$ and $y$ : $\qquad$
11. a number decreased by 8 : $\qquad$
12. four more than twice a number: $\qquad$
13. 5 decreased by a number: $\qquad$
14. twice the sum of a number and five: $\qquad$
15. the sum of the squares of $x$ and $y$ : $\qquad$
16. 4 more than triple a number: $\qquad$
17. the sum of twice a number and four: $\qquad$
18. Maggie earns $\$ 5$ per hour when she babysits 1 child. She earns $\$ 8$ per hour when she babysits 4 children. Let $x$ represent the number of hours she babysits 1 child and $y$ represent the number of hours she babysits 4 children. Create an expression that represents her total earnings.
Translate the following into algebraic equations, then solve. Let " $n$ " represent the number:
19. double a number is 14 : $\qquad$ $2 n=14, \quad n=7$
20. a number decreased by 6 is 5 : $\qquad$
21. one third of a number is 2 : $\qquad$
22. triple a number, increased by 2 is 8 : $\qquad$
23. six subtracted from five times a number is 9 : $\qquad$
24. half a number, less 5 is 12 : $\qquad$
25. twice a number, less 3 is 7 : $\qquad$
26. three more than twice a difference between a number and four is 9 : $\qquad$
27. two more than triple a number is 14 : $\qquad$
28. 5 less than one third a number is 1 : $\qquad$
29. quadruple a number decreased by 1 is 95 : $\qquad$

## Solving Word Problems

## 5 Steps ta saluing Ward Prablems:

1. Read the problem until you know what it is that the problem expects you to find
2. Identify what you're trying to find, and assign it a variable. DEFINE your variable with a "let" statement.
3. Write an equation following the instructions of the word problem. Use your list of math phrases to help you.
4. Solve the equation. This gives you the answer to the problem
5. Write a final "therefore" statement explaining what you've found.

## Examples:

## Ex 1

The number I'm thinking of plus two is three less than twenty five. What is my number?

Let $\qquad$ represent $\qquad$
Equation:
Therefore, my number is

## Ex 3

The length of a football field is 30 yards more than its width. If the perimeter is 260 m , what is the length and width?

Let $\qquad$
Equation:

## Ex 2

When I multiply Sophia's age by 5, I get the same as when I times her age by 2 and add 42 . How old is Sophia?

Let $\qquad$ represent $\qquad$
Equation:

Therefore, Sophia's age is

## Ex 4

In a triangle, the measure of the middle angle is triple the measure of the smallest angle, and the measure of the largest angle is $55^{\circ}$ greater than the measure of the smallest angle. Find the measures of the angles.
$\qquad$

## Practice Solving Word Problems

*Instructions: Solve the following on a separate sheet of paper. Remember to state your variables using "Let" statements and write concluding statements.

1. A square and a rectangle have the same perimeter. Find the side lengths of each figure.

2. Katherine is 2 years older than Christine. The sum of their ages is 16 .
a) Write an algebraic expression for each girl's age.
b) Write an equation to represent the sum of their ages.
c) How old is each girl?
3. The length of a rectangle is triple its width. The perimeter of the rectangle is 40 cm . What are the length and width?
4. Mary ran twice as far as Brenda. They ran a total distance of 12 km . Find how far each girl ran.
5. Katherine is 2 years older than Christine and Sarah is 1 year younger than Christine. Their total ages are 37. How old is each girl?
6. Mr. Kupsch operates a variety store with two friends, Mr. Ma and Mr. Lambert. Ma earns twice as much as Lambert. Kupsch earns $\$ 200 /$ week more than Ma. Their total weekly payroll is $\$ 1450$. How much does each person earn?
7. The length of a rectangular pool is 12 m greater than the width. What are the length and width of the pool if the perimeter of the pool is 96 m ?
8. The length of a rectangle is triple its width. The perimeter of the rectangle is 40 cm . What are the length and width?
9. The sum of two consecutive integers is 47. Find the integers.
10. Sarah works at a hockey arena selling water bottles. She is paid $\$ 6 / \mathrm{h}$ plus $\$ 0.50$ commission for every bottle sold.
a) Find Sarah's earnings (E) if she sells 42 water bottles during a 4-hour shift.
b) How many water bottles must she sell to earn $\$ 100$ in 7 hours?
11. Jason is twice as old as Michael. The sum of their ages three years ago was 45 . What are their ages now?
12. The sum of two consecutive integers is 47 .
a) Let $x$ represent the lesser integer. Write an algebraic expression to represent the greater integer.
b) Write an equation to represent the sum of the integers.
c) Find the integers.
13. The sum of three consecutive odd integers is 57.
a) Let $x$ represent the least integer. Write an algebraic expression to represent each of the other integers.
b) Write an equation to represent the sum of the integers.
c) Find the integers.
14. Three consecutive even integers have a sum of 102.
a) Write an algebraic expression to represent each integer.
b) Write an equation to represent the sum of the integers.
c) Find the integers.
15. Christine is looking to hire an electrician. Crazy Circuit charges $\$ 30$ per hour, plus $\$ 150$ to make the house call.
a) Write an equation relating total cost, and number of hours.
b) Using your equation, find how much it would cost if Christine hired the electrician for 4 hours.


To rearrange a formula, you need to use opposite operations to isolate a chosen variable.

| Examples: | Now you try: |
| :--- | :--- |
| Ex 1. Isolate $\mathbf{b}$ in $\mathbf{d}=\mathbf{m t} \mathbf{+ b}$ | Ex 2. Isolate $\mathbf{y}$ in $\mathbf{5 x} \mathbf{+} \mathbf{y}=\mathbf{3}$ |
|  |  |
| Ex 3. Rearrange to isolate $\mathbf{m}$ in $\mathbf{y}=\mathbf{m x}+\mathbf{b}$ | Ex 4. Rearrange to isolate $\mathbf{x}$ in $\mathbf{y}=\mathbf{m x}+\mathbf{b}$ |
| Ex 5. Isolate $\mathbf{t}$ in $\mathbf{v}=\frac{\mathbf{d}}{\mathbf{t}}$ |  |

## Solving Equations with ONE Fraction

1. Clear a fraction by multiplying BOTH sides of the equation by the denominator.
2. If there is more than one fraction, clear them by multiplying both sides of the equation by the LCD (lowest common denominator)

Examples:

| Ex 1 Solve for a. $\frac{a}{4}=-3$ | Ex 2 Solve for $b$. $\frac{1}{4} b=5$ |
| :---: | :---: |
| Ex 3 Solve for c . $-6=\frac{3 c}{5}$ | Ex 4 Solve for d. $\frac{4 d+2}{-3}=6$ |

## Practice Solving Equations with ONE Fraction

1. $\frac{t}{-3}=6$
2. $\frac{-2}{7} x=6$
3. $-5=\frac{-4 x}{6}$
4. $\frac{-3 m-8}{8}=-5$

## Solving Equations with TWO Fractions

1. Clear a fraction by multiplying BOTH sides of the equation by the denominator.
2. If there is more than one fraction, clear them by multiplying both sides of the equation by the LCD (lowest common denominator)


| Ex 7 Solve for g. $\frac{2}{5}(g-2)=-3$ | Try these: <br> a) $\frac{2}{3}(3 x+1)=5$ | b) $\frac{3}{4}(2 x+1)=2$ | c) $\frac{3}{4}(2 x+1)=\frac{2}{3}(x-5)$ |
| :---: | :---: | :---: | :---: |
| Ex 8 Solve for h . $\left(\frac{k+2}{3}\right)=\left(\frac{k-4}{5}\right)$ | d) $\left(\frac{3 x+4}{5}\right)=\left(\frac{x}{6}\right)$ | e) $\left(\frac{y+2}{9}\right)=\left(\frac{y-3}{3}\right)$ | f) $\left(\frac{3 c-2}{4}\right)=\left(\frac{2 c-1}{3}\right)$ |

## Word Problems with Fractions and Rearranging

1. Mr. Tan is designing a Japanese rock garden in the shape of a right angle triangle so that the second shortest side is twice the length of the shortest side. The area of the garden must be $30 \mathrm{~m}^{2}$. What are the dimensions of all three sides?
2. The perimeter of the small square is one-third the perimeter of the large square. What are the side lengths of the squares?

3. The ages of Marty and John total 27 years. Marty's age plus twice John's age is 40 . How old is each person?
4. Bill is three years younger than Dan. Eight years ago, Bill was one-half of Dan's age. How old is each boy now?
