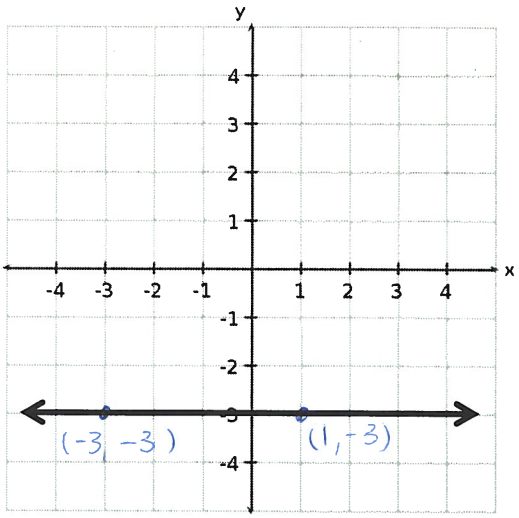
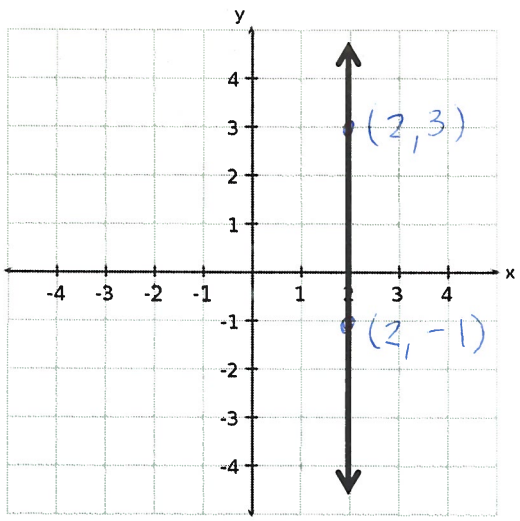
Horizontal and Vertical Lines

9. Calculate the slope of the line pictured below



$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-3)}{-3 - 1} = \frac{0}{-4} = 0$

10. Calculate the slope of the line pictured below



$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-1)}{2 - 2} = \frac{4}{0}$ = undefined

Conclusions:

Horizontal Lines...

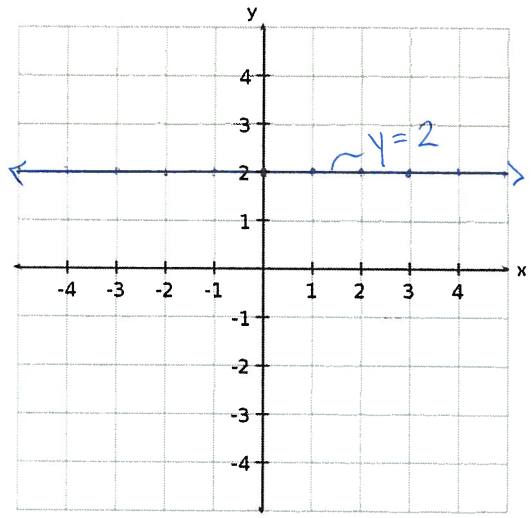
always have a slope of 0

Vertical Lines...

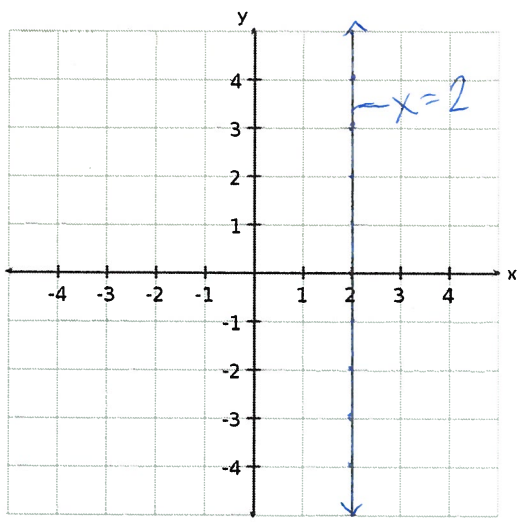
always have a slope that is undefined

http://www.mathsisfun.com/flash.php?path=%2Falgebra/images/line-to-equation.swf&w=600&h=600&col=%23FFFFFF&title=Equation+of+a+Line+from+2+Points

11. Graph the line y = 2



12. Graph the line x = 2



Conclusions:

Equations of **Horizontal** Lines...

always take the form: $y = \_\_\_$

↑ some number

Equations of **Vertical** Lines...

always take the form: $x = \_\_\_$

↑ some number

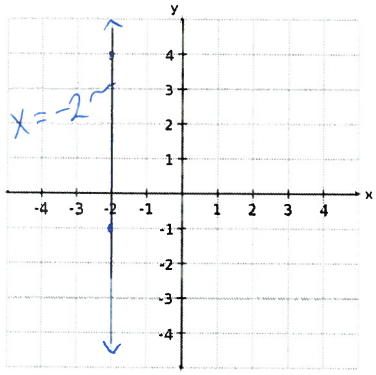
A **horizontal** line has a slope of **0**

A **vertical** line has an **undefined** slope

Practice 8 – Horizontal and Vertical Lines

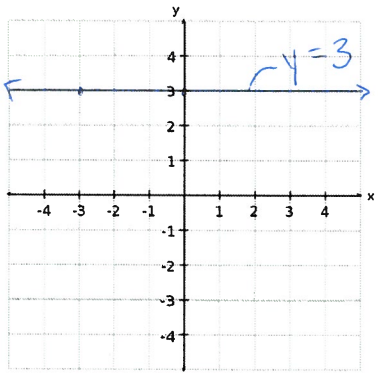
22. Graph each of the following lines, determine the slope, and determine the equation:

Example 1: Passes through (–2, 4) and (–2, –1)



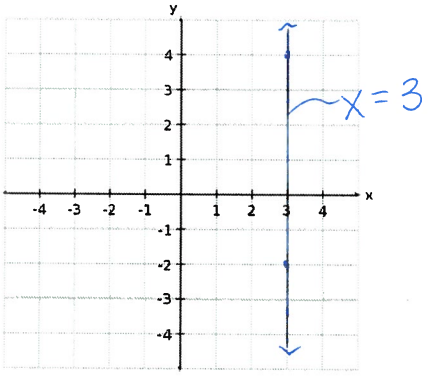
m = undefined Equation = $x = -2$

Example2: Passes through (0, 3) and (–3, 3)



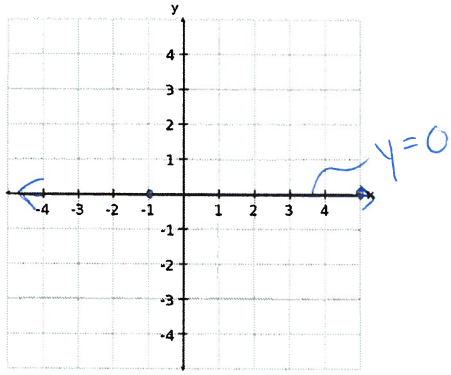
m = 0 Equation = $y = 3$

a. Passes through (3, 4) and (3, –2)



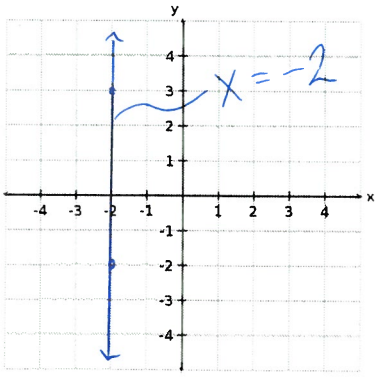
m = undefined Equation = $x = 3$

b. Passes through (–1, 0) and (5, 0)



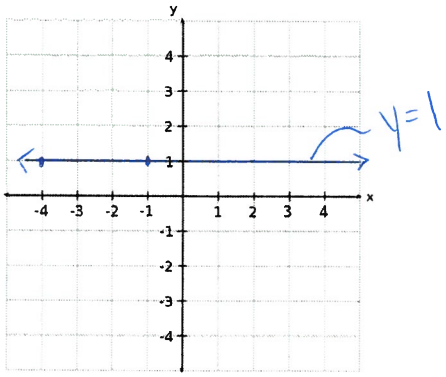
m = 0 Equation = $y = 0$

c. Passes through (–2, –2) and (–2, 3)



m = undefined Equation = $x = -2$

d. Passes through (–4, 1) and (–1, 1)



m = 0 Equation = $y = 1$

23. Determine the slope of each of the following lines (Hint – you do not NEED to graph):
    a. Passes through (3, –4) and (–8, –4)

    $m = 0$

    b. Passes through (5, –4.8) and (5, –3.9)

    $m =$ undefined

    c. Passes through (–7.2, 9.2) and (–3.5, 9.2)

    $m = 0$

    d. Passes through (3.5, –3.5) and (3.5, 3.5)

    $m =$ undefined.

24. Determine the equation of each of the following lines (Hint – you do not NEED to graph):
    a. Passes through (0, 4) and (–5, 4)

    $y = 4$

    b. Passes through (–2, 8.4) and (–2, 90)

    $x = -2$

    c. Passes through (3.5, 5.6) and (3.5, 1.2)

    $x = 3.5$

    d. Passes through (9.3, 6.7) and (–9.3, 6.7)

    $y = 6.7$

    e. Slope is undefined and it passes through (5,6)

    $x = 5$

    f. Passes through (–3,4) and has a slope of 0

    $y = 4$

    g. Is horizontal and passes through (3,2)

    $y = 2$

    h. Is vertical and passes through (–83.4, –17.2)

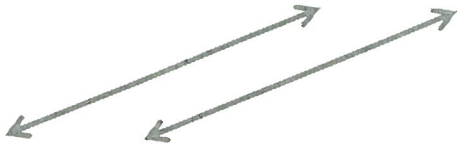
    $x = -83.4$

V H N

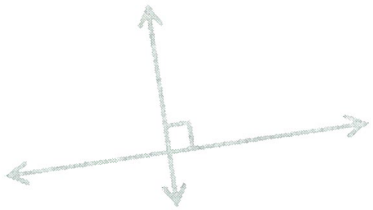
25. Determine whether the following lines are vertical, horizontal or neither:
    H a. y = 5
    V b. x = –3.4
    V c. Passes through (7.8, 4) and (7.8 –4)
    N d. Passes through (5.73, 4.02) and (6.73, –4.02)
    N e. Passes through (6.5, 4.2) and (–6.5, –4.2)
    H f. y = 45

Reminders:

Parallel lines are two lines that will go on forever and never meet



Perpendicular lines meet at a right angle (90°)



A reciprocal (or multiplicative inverse) is when you take a fraction and flip it upside down

For example, the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$ or 2

Mini Practice:

Number	Reciprocal
7 = $\frac{7}{1}$	$\frac{1}{7}$
$\frac{2}{3}$	$\frac{3}{2}$

A negative reciprocal is when you take the reciprocal of a number, and then change the sign

For example, the negative reciprocal of $\frac{1}{2}$ is $\frac{-2}{1}$ or −2

To find a negative reciprocal:

1. Write the number as a **fraction**
2. **Flip the fraction** upside down (switch the numerator and the denominator)
3. **Change the sign** of the fraction. If it is positive, make it negative. If it is negative, make it positive.

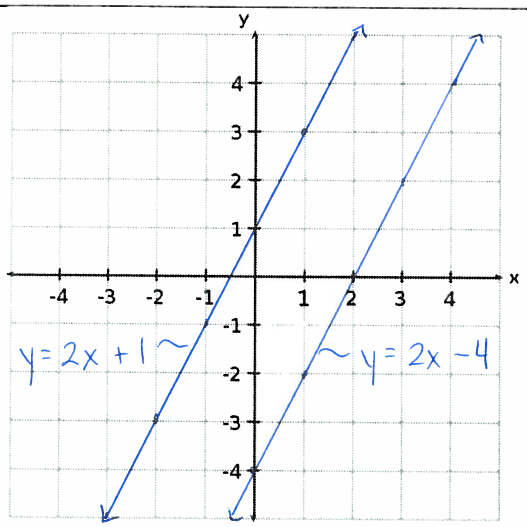
Mini Practice:

Number	Reciprocal	Negative Reciprocal
5 = $\frac{5}{1}$	$\frac{1}{5}$	$\frac{-1}{5}$
−6 = $\frac{-6}{1}$	$\frac{1}{-6}$	$\frac{1}{6}$
$\frac{1}{5}$	$\frac{5}{1}$	$\frac{-5}{1} = -5$
$\frac{-3}{4}$	$\frac{4}{-3}$	$\frac{4}{3}$
13.4 = $\frac{13.4}{1}$	$\frac{1}{13.4}$	$\frac{-1}{13.4} = -0.0746$ or $\frac{-1}{13.4} \times \frac{5}{5} = \frac{-5}{67}$
−1 = $\frac{-1}{1}$	$\frac{1}{-1}$	1

Parallel and Perpendicular Lines

13. Graph the following lines on the same grid:

$y = 2x + 1$ and $y = 2x - 4$



What do you notice about the way these two lines look on the grid?

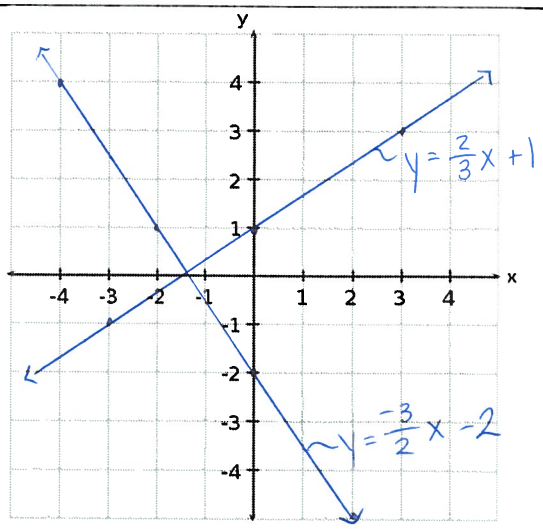
They are parallel

What do you notice about the SLOPES of the two lines?

They are the same.

14. Graph the following lines on the same grid:

$y = \frac{2}{3}x + 1$ and $y = \frac{-3}{2}x - 2$



What do you notice about the angle that these two lines cross at?

They cross at a 90° angle.
They are perpendicular.

What do you notice about the SLOPES of the two lines?

slope of line 1 = $\frac{2}{3}$ → (negative reciprocal) → $\frac{-3}{2}$ ↖ this is the same as slope of line 2!

They are negative reciprocals

Conclusions:

Parallel lines...

Have the same slope

Perpendicular lines...

Have slopes that are negative reciprocals

Practice 9 – Parallel and Perpendicular Lines

26. For each of the following, find the equation of a line that is parallel, **and** a line that is perpendicular:

*Since "b" can be any number, your answer may be different

		Parallel	Perpendicular
a.	**Example 1: y = –4x + 2**	$y = -4x + 1$ or "b" $y = -4x + 3$ or can be any number $y = -4x + 4$...	$-4 = \frac{-4}{1} \rightarrow \frac{1}{4}$ $y = \frac{1}{4}x + 1$, $y = \frac{1}{4}x + 2$, $y = \frac{1}{4}x + 3$...
b.	**Example 2: y = 5** (this line is horizontal)	$y = 1$ $y = 2$ $y = 3$... any horizontal line	$x = 1$ $x = 2$ $x = 3$... any vertical line
c.	y = –3x +4	$y = -3x + 20$	$y = \frac{1}{3}x - 25$
d.	$y = \frac{2}{3}x - 5$	$y = \frac{2}{3}x - 42$	$y = \frac{-3}{2}x + 1$
e.	A line that passes through (4, 5) and (–7, 2) $m = \frac{5-2}{4+7} = \frac{3}{11}$ $y = \frac{3}{11}x + b$ $5 = \frac{3}{11}(4) + b$ $\frac{43}{11} = b$ $y = \frac{3}{11}x + \frac{43}{11}$	$y = \frac{3}{11}x - 12$	$y = \frac{-11}{3}x + 15.2$
f.	A line that passes through (–3, –2) and (–1, –6) $m = \frac{-2+6}{-3+1} = \frac{4}{-2} = -2$ $y = -2x + b$ $-6 = -2(-1) + b$ $-6 = 2 + b$ $-8 = b$ $y = -2x - 8$	$y = -2x - 9$	$y = \frac{1}{2}x$
g.	y = –4	$y = 10$	$x = 10$
h.	x = 3	$x = 2$	$y = 18$
i.	y = 4x – 2.5	$y = 4x + 3$	$y = \frac{-1}{4}x - 7$
j.	$y = \frac{-1}{2}x = 10$	$y = \frac{-1}{2}x + 13$	$y = 2x - 19$

k.	A line that passes through (0, –3) and (9, 17) $m = \frac{17+3}{9-0} = \frac{20}{9}$ $y = \frac{20}{9}x + b$ $-3 = \frac{20}{9}(0) + b$ $-3 = b$ $y = \frac{20}{9}x - 3$	$y = \frac{20}{9}x + 4$	$y = \frac{-9}{20}x - 1$
l.	A line that passes through (4.5, 6.7) and (2.3, –4.1) $m = \frac{6.7+4.1}{4.5-2.3} = \frac{10.8}{2.2} = \frac{54}{11}$ $y = \frac{54}{11}x + b$ $6.7 = \frac{54}{11}(4.5) + b$ $-\frac{169.3}{11} = b \quad -15.39 = b$ $y = \frac{54}{11}x - 15.39$	$y = \frac{54}{11}x - 2$	$y = \frac{-11}{54}x + 9$

27. Conclude whether the following pairs represent parallel lines, perpendicular lines, or neither, and state WHY:

Example 3: **$y = 8x - 15$** **and** **$y = -0.125x + 3$**

Not parallel since $8 \neq -0.125$ $\quad 8 = \frac{8}{1} \rightarrow \frac{-1}{8} = -0.125$

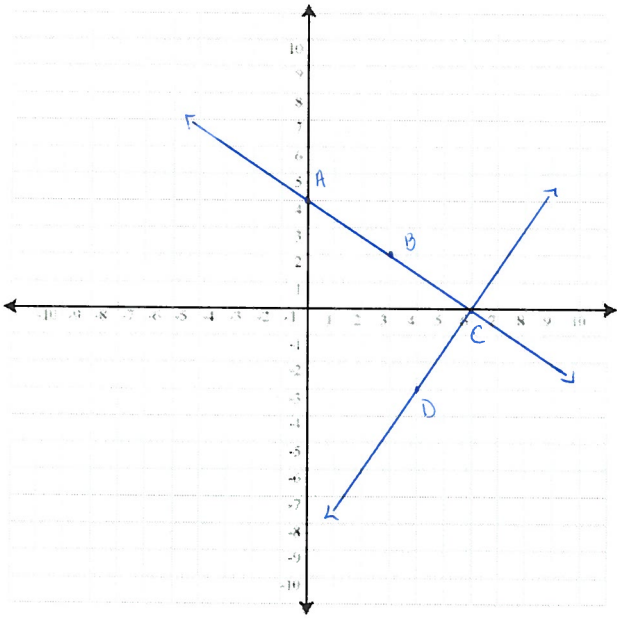
They are parallel since negative reciprocal of 8 is -0.125

a.	$y = \frac{1}{4}x - 3$ and $y = \frac{1}{3}x - 4$ Neither, since $\frac{1}{4} \neq \frac{1}{3}$, & the negative reciprocal of $\frac{1}{4}$ is -4, not $\frac{1}{3}$	d.	$y = 4x$ and $y = \frac{1}{4}x + 8$ Neither, since $4 \neq \frac{1}{4}$, & the negative reciprocal of 4 is $-\frac{1}{4}$, not $\frac{1}{4}$
b.	$y = -3x + 10$ and $y = \frac{1}{3}x - 14$ Perpendicular, since the negative reciprocal of -3 is $\frac{1}{3}$	e.	$y = 3x + 2$ and $y = -3x - 1$ Neither, since $3 \neq -3$, & the negative reciprocal of 3 is $-\frac{1}{3}$, not -3.
c.	$y = 0.5x - 13$ and $y = \frac{1}{2}x + 6$ Parallel, since $0.5 = \frac{1}{2}$	f.	$y = -2x + 12$ and $y = 13 - 2x$ Parallel, since $-2 = -2$.

Using parallel and perpendicular lines to analyze lines and triangles:

Example 4: Line AB passes through A(0,4) and B(3,2). Line CD passes through C(6,0) and D(4, –3).

a. Plot these points and draw the lines.



b. Do these lines appear to be parallel, perpendicular, or neither? Explain.

Perpendicular. They appear to meet at a 90° angle.

c. Find the slopes of the line segments.

AB → $m = \frac{4-2}{0-3} = \frac{2}{-3}$

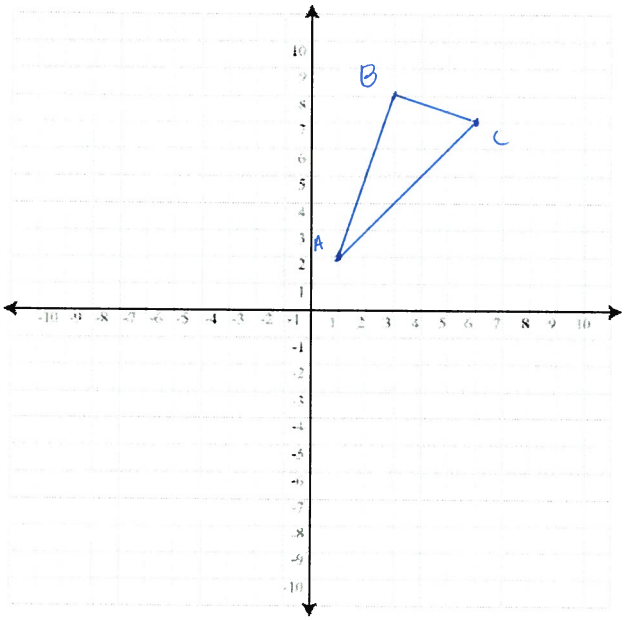
CD → $m = \frac{-3-0}{4-6} = \frac{-3}{-2} = \frac{3}{2}$

d. Explain how the slopes can be used to conclude whether these lines are parallel, perpendicular, or neither.

Take the slope of AB and find the negative reciprocal: $\frac{2}{-3} \rightarrow \frac{-3}{2} \rightarrow \frac{3}{2}$
Since the negative reciprocal of AB is the same as CD, the two lines are perpendicular.

28. A triangle has vertices A (1, 2), B(3, 8) and C(6, 7).

a. Plot these points and draw the triangle.



b. Does this appear to be a right triangle? Explain.

Yes. ∠ABC appears to be 90°

c. Find the slopes of the line segments that form this triangle.

AB → $m = \frac{3}{1}$

BC → $m = \frac{-1}{3}$

AC → $m = 1$

d. Explain how the slopes can be used to conclude whether or not this is a right triangle. Is it?

If any 2 slopes are negative reciprocals of eachother, then it is a right triangle.
Since the slopes of AB and BC are negative reciprocals of eachother, this is a right triangle.

Standard Form

We've talked about equations of lines in Slope y-Intercept Form: **y = mx + b**. Sometimes we will see equations of lines in a different form, called **Standard Form**, which is:

$$Ax + By + C = 0$$

x and y are variables

A, B, and C are numbers

- A, B, and C are integers (positive or negative whole numbers)
- A cannot be negative
- A and B cannot ***both*** be zero

15. You are given the equation of a line in Standard Form, which is: **4x + 2y – 10 = 0**. What is the equation of this line in Slope y-Intercept Form?

To find the equation of this line in y = mx + b form, you need to **rearrange** to **isolate y**.

$$4x - 2y - 10 = 0$$

$$4x - 2y + 2y - 10 = 0 + 2y$$

$$\frac{4x - 10}{2} = \frac{2y}{2}$$

$$2x - 5 = y$$

$$\therefore y = 2x - 5$$

16. You are given the equation of a line in Slope y-Intercept Form, which is: **y = –3x + 5**. What is the equation of this line in Standard Form?

To find the equation of this line in y = mx + b form, you need to **rearrange** to **isolate 0**.

$$y = -\frac{2}{3}x + \frac{3}{4}$$

$$y - y = -\frac{2}{3}x + \frac{3}{4} - y$$

$$(-12)(0) = \left(-\frac{2}{3}x + \frac{3}{4} - y\right)(-12)$$

$$0 = 8x - 9 + 12y$$

$$0 = 8x + 12y - 9$$

$$\therefore 8x + 12y - 9 = 0$$

Practice 10 – Standard Form

29. The following equations are in Standard Form. Convert them to Slope y-Intercept Form.

a. $5x + 6y + 12 = 0$

$y = \frac{-5}{6}x - 2$

b. $7x - 2y + 1 = 0$

$y = \frac{7}{2}x + \frac{1}{2}$

c. $x - 5y - 2 = 0$

$y = \frac{1}{5}x - \frac{2}{5}$

d. $9x + 4y = 0$

$y = \frac{-9}{4}x$

e. $8x + 3y - 13 = 0$

$y = \frac{-8}{3}x + \frac{13}{3}$

f. $9x + 17y - 14 = 0$

$y = \frac{-9}{17}x + \frac{14}{17}$

g. $16x + 34y + 300 = 0$

$y = \frac{-8}{17}x - \frac{150}{17}$

h. $x - y = 0$

$y = x$

i. $15x + 35y - 70 = 0$

$y = \frac{3}{7}x + 2$

j. $4x + y - 17 = 0$

$y = -4x + 17$

30. The following equations are in Slope y-Intercept Form. Convert them to Standard Form.

a. $y = 12x - 4$

$12x - y - 4 = 0$

b. $y = -3x - 1.5$

$3x + y + 1.5 = 0$

c. $y = \frac{3}{5}x - 4.6$

$6x - 10y - 45 = 0$

d. $y = \frac{2}{3}x - \frac{1}{2}$

$4x - 6y - 3 = 0$

e. $y = 4.5x - 13.2$

$45x - 10y - 132 = 0$

f. $y = -15x - 7$

$15x + y + 7 = 0$

g. $y = \frac{7}{8}x + \frac{3}{7}$

$49x - 56y + 24 = 0$

h. $y = 4x - \frac{2}{3}$

$12x - 3y - 2 = 0$

i. $y = -\frac{4}{5}x - \frac{9}{10}$

$8x + 10y + 9 = 0$

j. $y = \frac{1}{3}x - \frac{7}{9}$

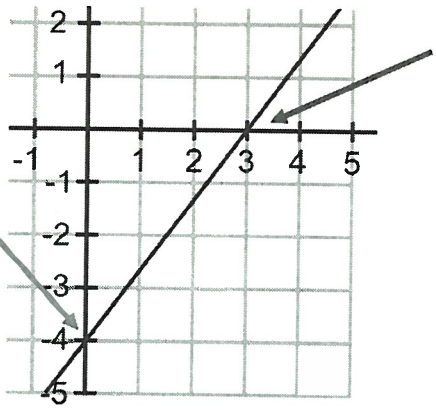
$3x - 9y - 7 = 0$

Graphing from Standard Form

The y-intercept of an equation is the point where a line crosses the y-axis. It is also known as "b"

A y-intercept always has an x coordinate of 0, so it takes the form (0, y)

The coordinates of the y-intercept of this line is (0, –4)



The other intercept is called the x-intercept

An x-intercept always has a y coordinate of 0, so it takes the form (x, 0)

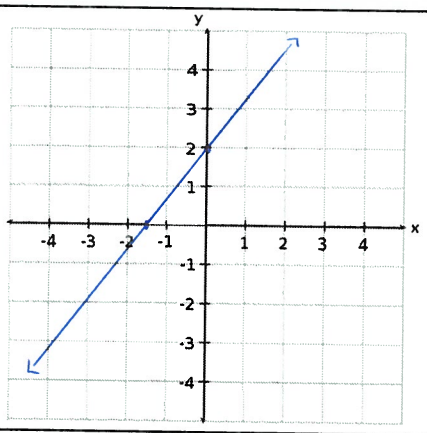
The coordinates of the x-intercept of this line is (3, 0)

17. Graph a line that has a **y-intercept of 2** and an **x-intercept of –1.5**.

What two points are you given?

(0 , 2) and (–1.5, 0)

y-intercept x-intercept



18. Given the equation **2x – 3y + 6 = 0**, graph the line by finding and plotting the x-intercept and the y-intercept

For your **y-intercept**, you know that **"x" is 0**

2x – 3y + 6 = 0

$2(0) - 3y + 6 = 0$

$-3y + 6 = 0$

$-3y + 3y + 6 = 0 + 3y$

$\frac{6}{3} = \frac{3y}{3} \rightarrow 2 = y$ (0, 2)

For your **x-intercept**, you know that **"y" is 0**

2x – 3y + 6 = 0

$2x - 3(0) + 6 = 0$

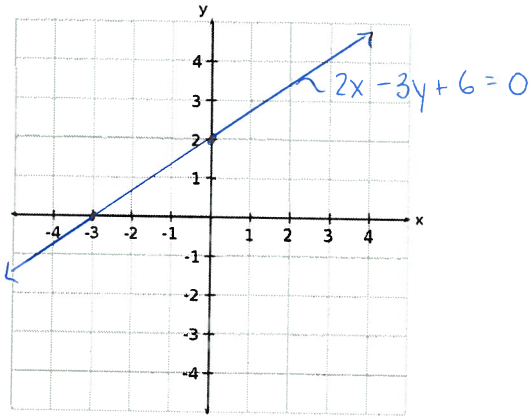
$2x + 6 = 0$

$2x + 6 - 6 = 0 - 6$

$\frac{2x}{2} = \frac{-6}{2}$

$x = -3$

(–3, 0)



Practice 11 – Graphing from Standard Form

31. The following equations are in Standard Form. Graph each line on the grid at the right by finding the x and y intercepts. LABEL each line.

a. $2x - y + 5 = 0$

(0, 5) ; (-2.5, 0)

b. $2x - 3y - 12 = 0$

(0, -4) ; (6, 0)

c. $x + 18y - 9 = 0$

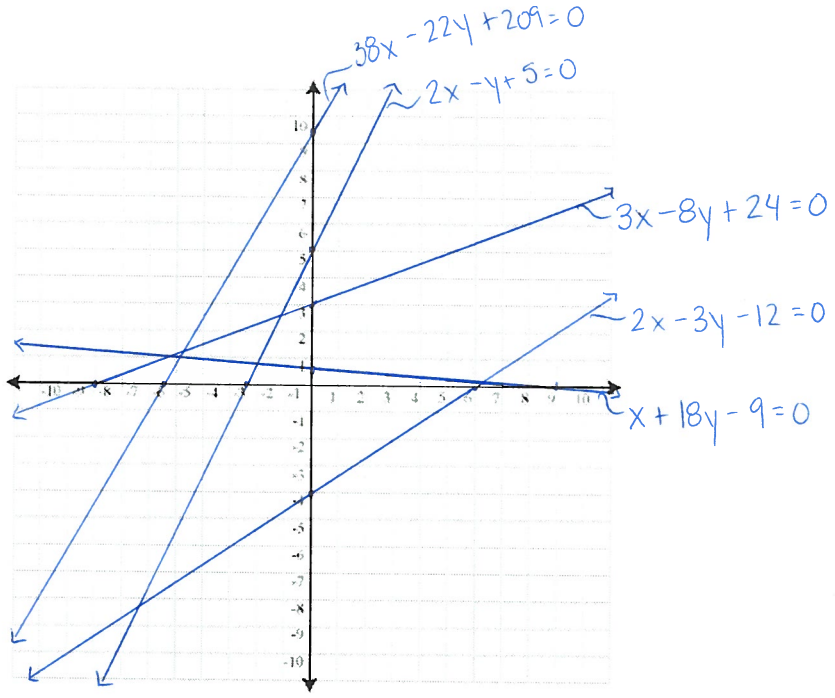
(0, 0.5) ; (9, 0)

d. $38x - 22y + 209 = 0$

(0, 10) ; (-5.5, 0)

e. $3x - 8y + 24 = 0$

(0, 3) ; (-8, 0)



32. The following equations are in Standard Form. Graph each line on the grid at the right by finding the x and y intercepts. LABEL each line.

a. $4x - y - 8 = 0$

(0, -8) ; (2, 0)

b. $15x - 4y + 30 = 0$

(0, 7.5) ; (-2, 0)

c. $7x - 4y - 28 = 0$

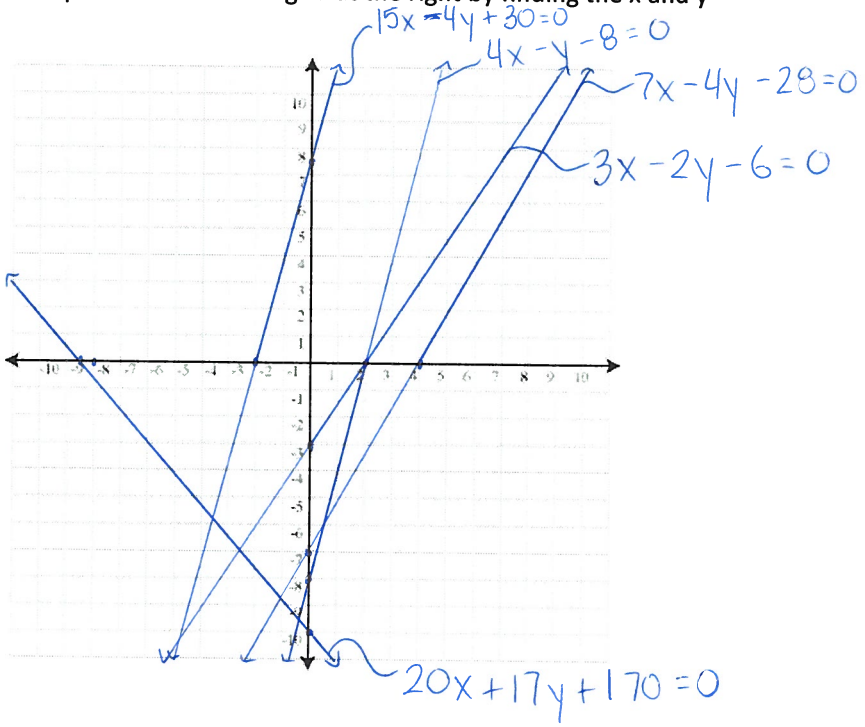
(0, -7) ; (4, 0)

d. $3x - 2y - 6 = 0$

(0, -3) ; (2, 0)

e. $20x + 17y + 170 = 0$

(0, -10) ; (-8.5, 0)



Word Problems in Standard Form

When we've written equations from word problems in the past, we've typically selected an independent variable and a dependent variable, then written the equation in y = mx + b form (Slope y-Intercept Form). Word problems that have variables where there is **NOT a dependent variable and an independent variable** are best written in **Standard Form**.

19a. You are in charge of buying the burgers and hotdogs for a party. You have $60 to spend. The burgers costs $3 each and the hotdogs are $2 each. Write an equation that relates the number of burgers and the number of hotdogs that you can buy.

Define your variables: Let b be number of burgers
Let h be number of hotdogs

Create your equation: $3b + 2h = 60$

19b. Convert your equation to Standard Form.

$3b + 2h = 60$
$3b + 2h - 60 = 0$

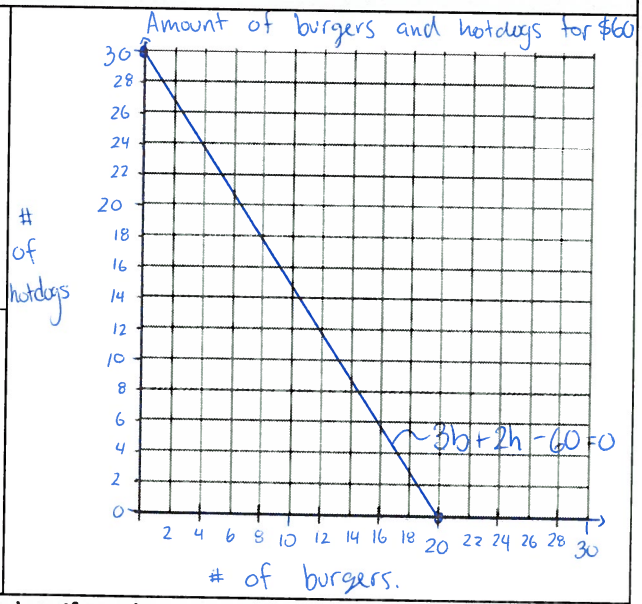
19c. Graph this relationship by finding and plotting the x- and y-intercepts.

Find your y-intercept: (# of hotdogs for 0 burgers)

$3b + 2h = 60$
$3(0) + 2h = 60$
$\frac{2h}{2} = \frac{60}{2}$
$h = 30$ $(0, 30)$

Find your x-intercept: (# of burgers for 0 hotdogs)

$3b + 2h = 60$
$3b + 2(0) = 60$
$\frac{3b}{3} = \frac{60}{3}$
$b = 20$ $(20, 0)$



19d. Using your EQUATION, determine how many hotdogs you can buy if you buy 15 burgers.

$3b + 2h = 60$
$3(15) + 2h = 60$
$45 + 2h = 60$
$\frac{2h}{2} = \frac{15}{2}$ $h = 7.5$

∴ You could buy 7 hotdogs (with money left over).

Practice 12 – Word Problems in Standard Form

33. You are buying $48 worth of lawn seed that consists of two types of seed. One type is Ryegrass that costs $4 per pound, and the other type is Bluegrass that costs $6 per pound.

a. Write an equation that represents the different amounts of Ryegrass and Bluegrass you can buy. Define your variables.

Define your variables:

Let r **be** # of pounds of Ryegrass

Let b **be** # of pounds of Bluegrass

Create your equation:

$4r + 6b = 48$

b. Convert your equation to standard form.

Ax + By + C = 0

$4r + 6b = 48$

$4r + 6b - 48 = 0$

*Note: It doesn't matter whether "r" or "b" is your "y"

c. Graph this relationship by finding and plotting the x- and y-intercepts.

Find your y-intercept:

$4(0) + 6b = 48$

$\frac{6b}{6} = \frac{48}{6}$

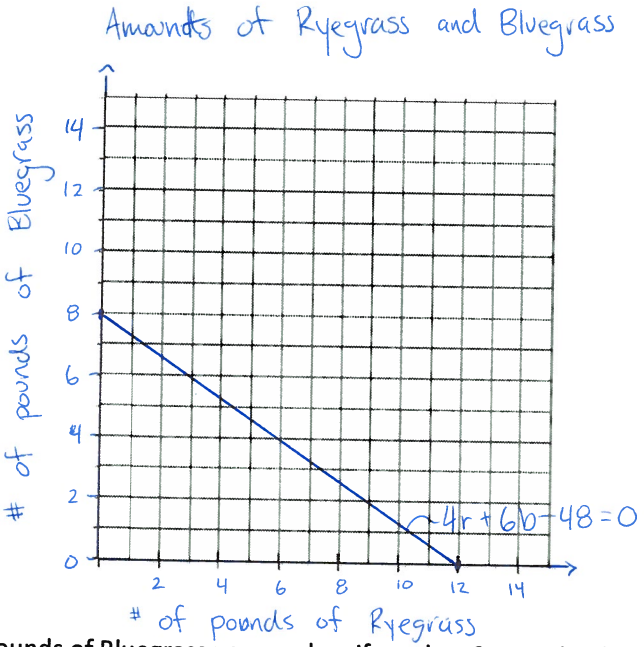
$b = 8$

Find your x-intercept:

$4r + 6(0) = 48$

$\frac{4r}{4} = \frac{48}{4}$

$r = 12$



d. Using your equation, determine how many pounds of Bluegrass you can buy if you buy 3 pounds of Ryegrass.

$4r + 6b = 48$

$4(3) + 6b = 48$

$12 + 6b = 48$

$6b = 48 - 12$

$\frac{6b}{6} = \frac{36}{6}$

$b = 6$

∴ You can buy 6 pounds of Bluegrass.

34. Your grandmother made 240 oz. of jelly. You have two types of jars. The first holds 10 oz. And the second holds 12 oz.

a. Write an equation that represents the different number of 10-oz. jars and 12-oz. jars that will hold all of the jelly. Define your variables.

Let f be the # of 10 oz jars.
Let s be the # of 12 oz jars.

$10f + 12s = 240$

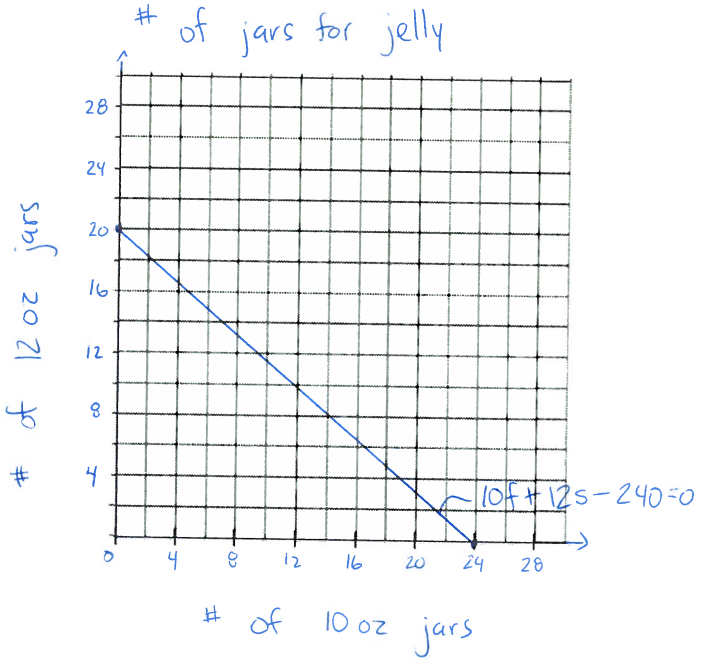
b. Convert your equation to standard form.

$10f + 12s - 240 = 0$

c. Graph this relationship by finding and plotting the x- and y-intercepts.

$f = 24$

$s = 20$



35. You are buying $30 worth of birdseed that consists of two types of seed. Thistle seed attracts finches and costs $2 per pound. Dark oil sunflower seed attracts many kinds of sunbirds and costs $1.50 per pound.

a. Write an equation that represents the different amounts of $2 thistle seed and $1.50 dark oil sunflower seed that you can buy. Define your variables.

Let t be amount of thistle seed
Let d be amount of dark oil sunflower seed

$2t + 1.50d = 30$

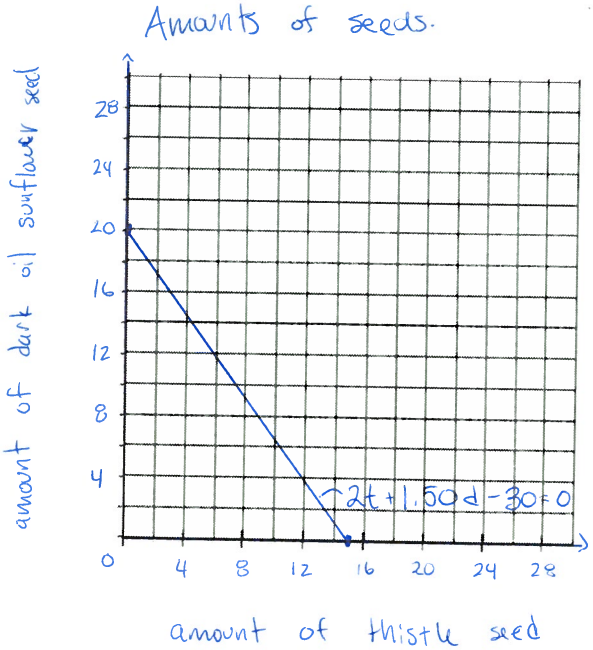
b. Convert your equation to standard form.

$2t + 1.50d - 30 = 0$

c. Graph this relationship by finding and plotting the x- and y-intercepts.

$t = 15$

$d = 20$



36. A 100-point test has *x* number of questions worth 2 points each and *y* number of questions worth 4 points each.

a. Write an equation that describes all possible numbers of questions that may be on the test. Define your variables.

Let x be # of 2 point questions

Let y be # of 4 point questions

$2x + 4y = 100$

b. Convert your equation to standard form.

$2x + 4y - 100 = 0$

c. If you have 24 questions worth 4 points each, how many questions will be worth 2 points? Use your equation to solve.

$2x + 4y = 100$

$2x + 4(24) = 100$

$x = 2$

∴ You will have 2 2-point questions.

d. Graph this relationship if you feel that you need extra practice graphing.

37. Louise has $36 in five-dollar bills and loonies.

a. Write an equation that represents how many of each denomination she could have. Define your variables.

Let f be # of five-dollar bills

Let c be # of loonies

$5f + c = 36$

b. Convert your equation to standard form.

$5f + c - 36 = 0$

c. If Louse has 2 five-dollar bills, how many loonies does she have? Use your equation.

$c = 26$ ∴ She has 26 loonies.

d. Graph this relationship if you feel that you need extra practice graphing.

38. The store at which Andy usually shops is having a sale. Roast beef costs $4 a pound and shrimp costs $10 a pound.

a. Write an equation to describe different possible combinations of roast beef and shrimp that he can buy for $96. Convert your equation to standard form.

let r be amount of roast beef
let s be amount of shrimp

$4r + 10s = 96 \rightarrow 4r + 10s - 96 = 0$

b. What is the greatest amount of shrimp he can buy? Use your equation.

$S = 9.6$ ∴ He can buy 9 pounds of shrimp.

c. Graph this relationship if you feel that you need extra practice graphing.

39. It will take 20 points to make the playoffs, the hockey team coach told the players. "We get 2 points for a win and 1 point for a tie."

a. Write an equation to describe the numbers of wins and ties that will let the team make the playoffs. Convert your equation to standard form.

let w be # of wins
let t be # of ties

$2w + t = 20 \rightarrow 2w + t - 20 = 0$

b. If the team wins 7 games, how many tie games will need to occur? Use your equation.

$t = 6$ ∴ They will need 6 tie games.

c. Graph this relationship if you feel that you need extra practice graphing.

40. The perimeter of a fenced rectangular area is 75 m.

a. Write a linear model relating the length and width. Convert your equation to standard form.

let l be length
Let w be width

$2l + 2w = 75 \rightarrow 2l + 2w - 75 = 0$

b. If the length is 5 m, find the width.

$w = 32.5$ ∴ The width is 32.5 m.

c. Graph this relationship if you feel that you need extra practice graphing.