Slope

Slope of an object

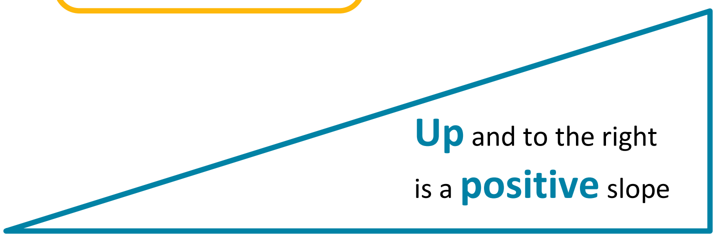
When we find the **slope** of an object, we are finding how **steep** it is.

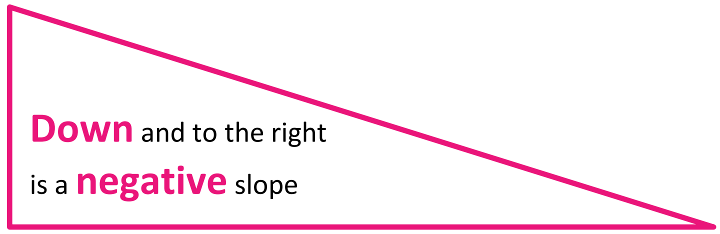
To find slope, we find the height of the object, and divide it by the length. Here it is in an equation:

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

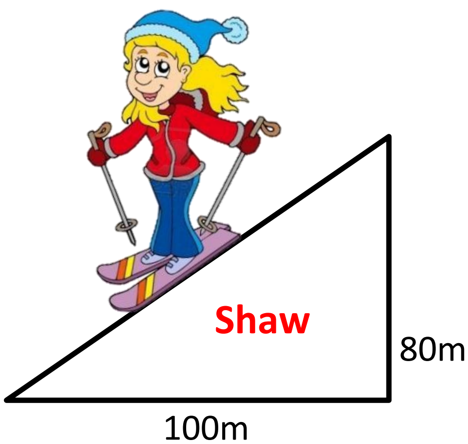
rise ← Height (vertical/up)

run ← Length (horizontal/right)

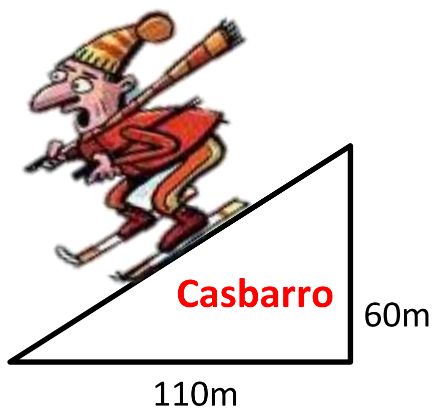




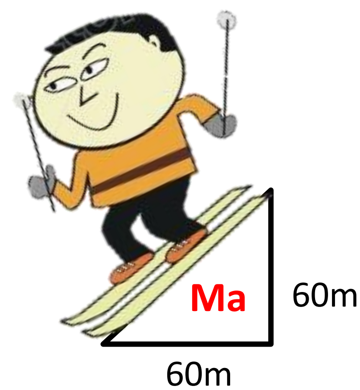
Ex 1. Mme. Shaw, Mr. Casbarro, and Mr. Ma all go skiing. Each takes a different run, and when they get back to the chalet, they argue over who is the biggest daredevil. Each insists that their ski run was the steepest. They know the heights and lengths of the ski runs. **Find who skied the steepest run.**



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

Ex 2. Building regulations state that wheelchair ramps cannot have a slope greater than $\frac{1}{12}$. If the height of a wheelchair ramp is 1.5m, what is the maximum length it can have?

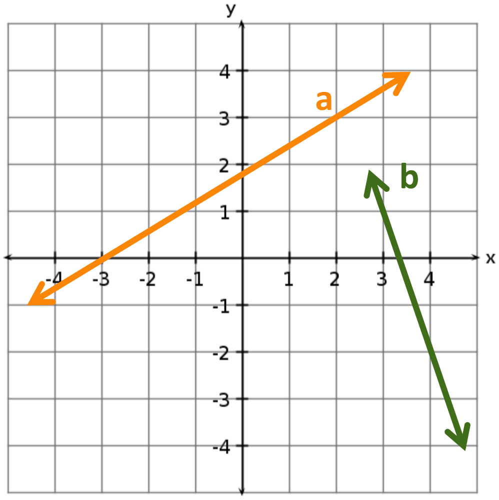
Slope of a Line

Slope **is sometimes represented by the letter "m"**

$$\textbf{Slope} = \mathbf{m} = \frac{\textbf{rise}}{\textbf{run}}$$

...from a graph

Ex 3. Find the slopes of each line.

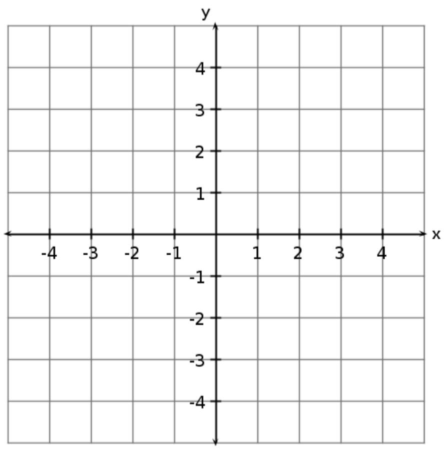


Slope of a

Slope of b

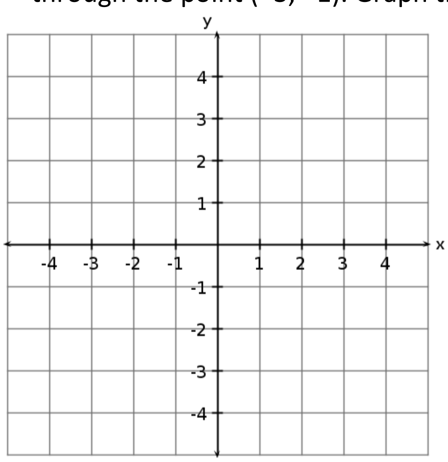
...from two points

Ex 4. What is the slope of the line that goes through points **C(–3, 1)** and **D(2,3)**?

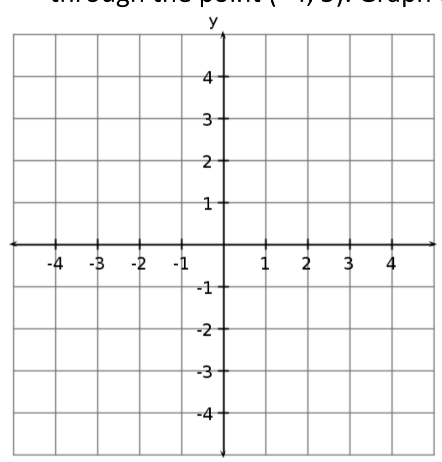


...graphing from a slope and a point

Ex 5. A line has a slope of $\frac{3}{4}$ and passes through the point (–3, –1). Graph this line.



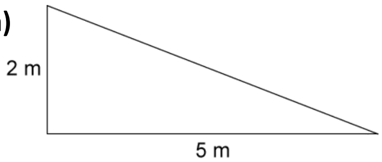
Ex 6. A line has a slope of $\frac{-2}{3}$ and passes through the point (–4, 5). Graph this line.



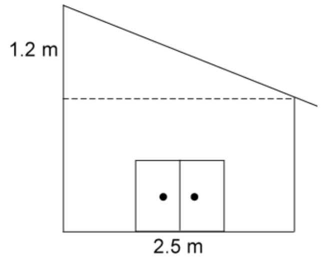
Practice 1 – Slope of Objects

1. Find the slope of each object/line segment.

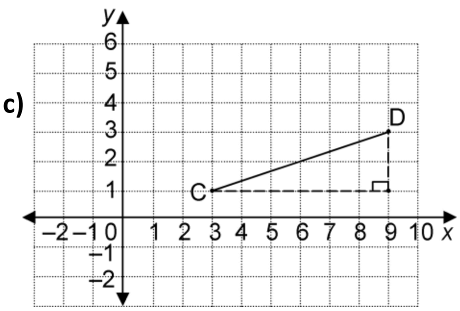
a)



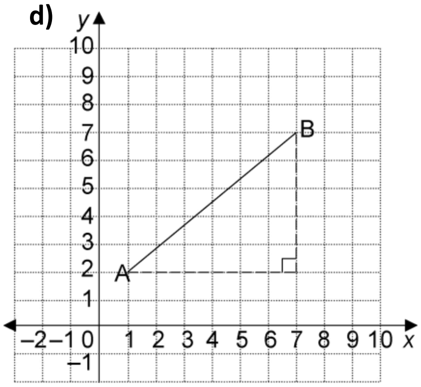
b)



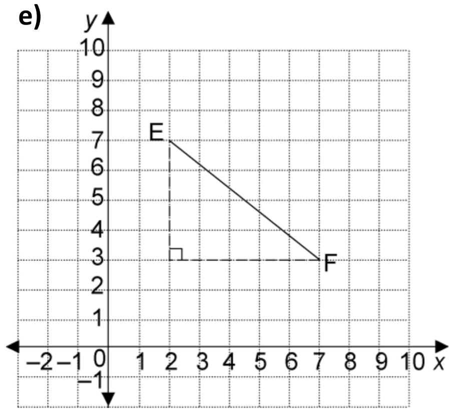
c)



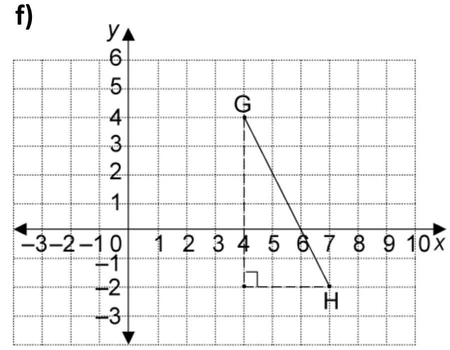
d)



e)



f)

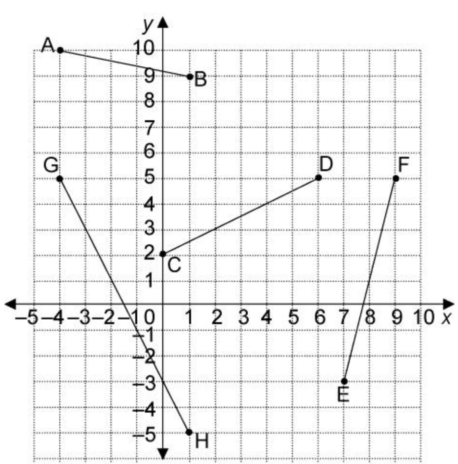


2. The slope of a roof is referred to as its **pitch**. Using a ruler, determine the pitch of each of these roofs.

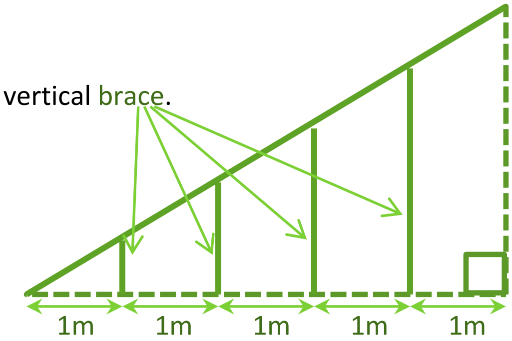
a)

b)

3. Find the slope of each line segment.



4. A ramp needs to have a slope of $\frac{3}{5}$. determine the length of each vertical brace.



5. A steel beam goes between the tops of two buildings that are 7m apart. One building is 41m tall. The other is 52m tall. What is the slope of the beam?

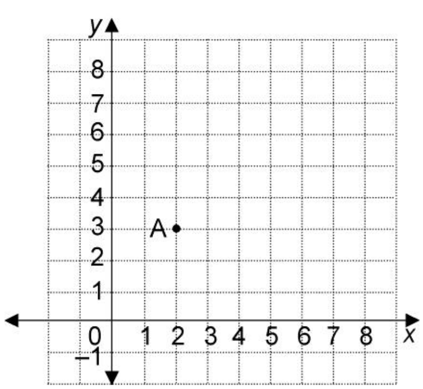
6. For safety reasons, an extension ladder should have a slope of between 6.3 and 9.5 when it is placed against a wall. If a ladder reachers 8m up a wall, what are the maximum and minimum distances from the foot of the ladder to the wall?

7. For safety, the slope of a staircase must be greater than 0.58 and less than 0.70.
A staircase has a vertical rise of 2.4 m over a horizontal run of 3.5 m.

a) Find the slope of the staircase.

b) Is the staircase safe?

8. Point A (2, 3) is plotted on the grid. Draw a line segment AB with slope $-\frac{1}{2}$. What are possible coordinates of B?



Slope as a rate of change

Slope represents the **steepness** of a line on a graph, but it is also a **rate of change**.

A **rate of change** is a change in one quantity relative to the change in another quantity.

When we represent a slope as a rate of change, we need to include **units**, such as kilometres per hour.

Here is the new way we can represent slope:

$$\textbf{Slope} = \frac{\Delta y}{\Delta x} = \frac{\textit{change in } y}{\textit{change in } x}$$

To calculate how a value changes, we take the end value, and from it, we subtract the beginning value. For example:

Jenna's weight increased from 107 lbs to 113 lbs. By how much did her weight **change**?

change = end value – beginning value
= 113 – 107
= 6 ∴ her weight changed by 6 lbs

So, if we had two "y" values, we would represent change in these values as $y_2 - y_1$

And if we had two "x" values, we would represent change in these values as $x_2 - x_1$

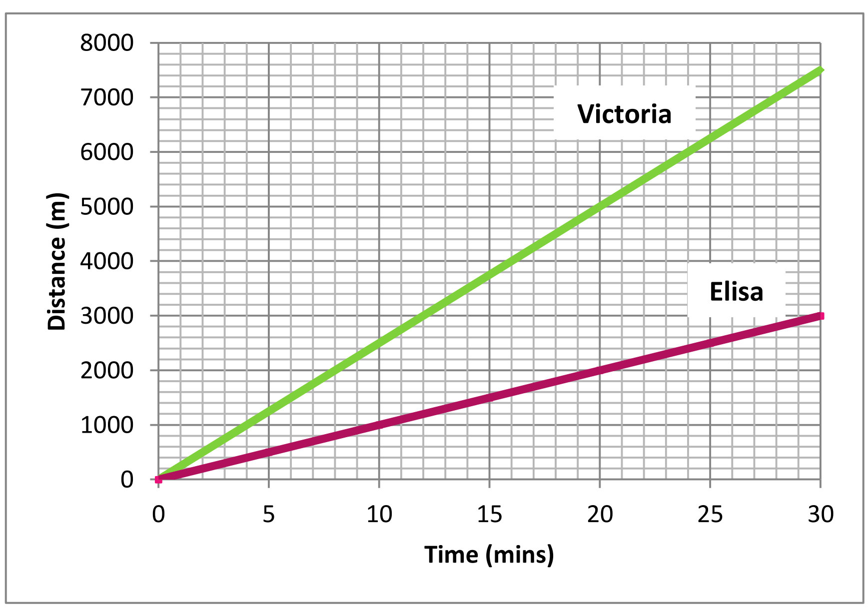
Where (x_1, y_1) and (x_2, y_2) are two points on the graph/in the table.

This means, that slope can be represented as follows:

$$\textbf{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope as a rate of change from a graph

Ex 7. Victoria and Elisa both run every morning. The following graph shows the distance they travelled over time.



a) Find the speed that each girl travelled (speed is calculated as $\frac{\text{distance}}{\text{time}}$).

	Victoria	Elisa
Choose two points on the graph	(x_1,y_1) and (x_2,y_2) (,) and (,)	(x_1,y_1) and (x_2,y_2) (,) and (,)
Plug them into the slope formula	$\frac{y_2 - y_1}{x_2 - x_1} =$ ________________	$\frac{y_2 - y_1}{x_2 - x_1} =$ ________________
Simplify the top and the bottom		
Divide, or leave as a fraction if not evenly divisible		

b) Who ran faster? What was her speed?

c) What does a faster speed look like on a graph?

Slope as a rate of change from a table of values

Ex 8. This table of values shows the volume of gasoline remaining in a car's tank. It forms a linear relationship.

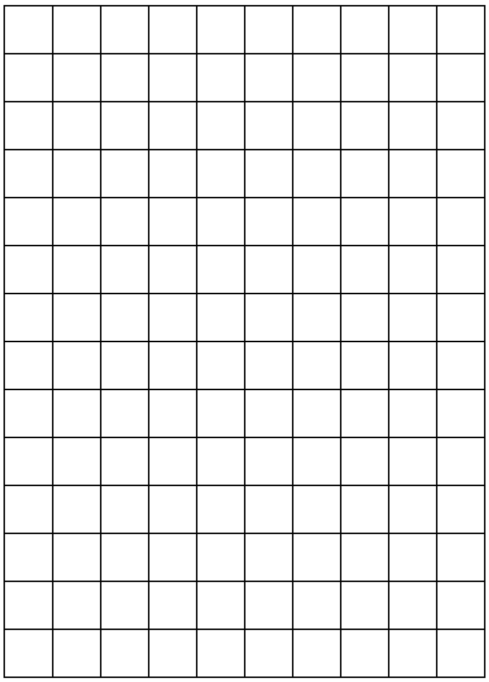
Distance (km)	0	100	200	300	400	500
Volume (L)	65	53	41	29	17	5

a) Calculate the slope of the line that would result from these data points.

Choose two points on the graph	
Plug them into the slope formula	
Simplify the top and the bottom	
Divide, or leave as a fraction if not evenly divisible	

b) Interpret the slope as a rate of change.

c) Graph this relationship.



d) What does a negative slope look like?

Homework: Pg 267 C1, C2

Pg 268 #1-10, 13, 18

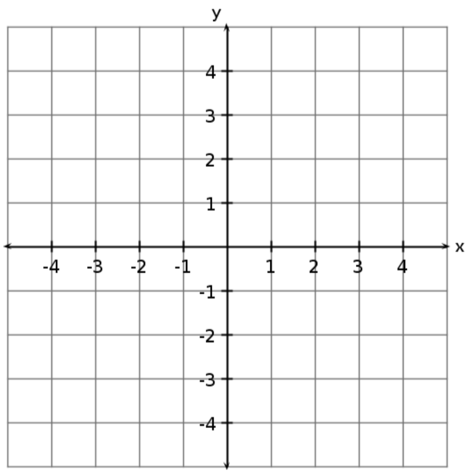
These are the different ways we can now say "slope"

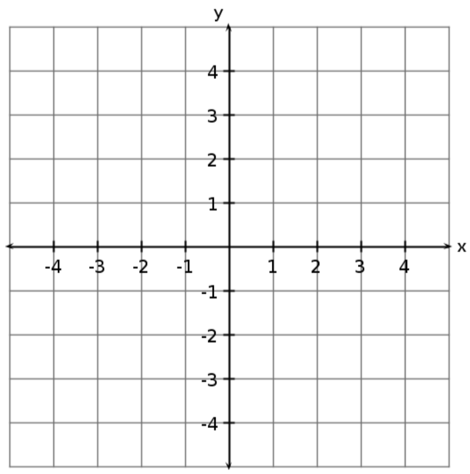
$$\textbf{Slope} = \textbf{m} = \frac{\textbf{rise}}{\textbf{run}} = \frac{\Delta y}{\Delta x} = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

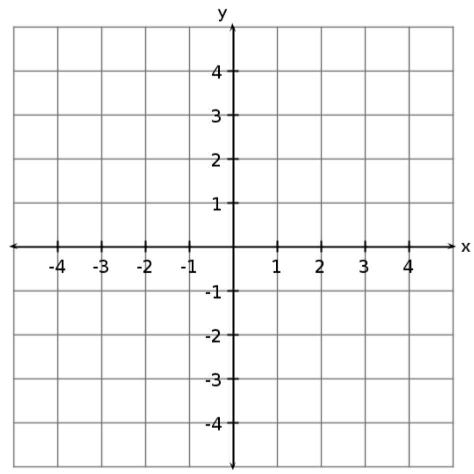
Practice 2 – Slope as a Rate of Change

9. Plot the following pairs of points on separate graphs, draw the line that passes through them, and calculate the slope using the slope formula.

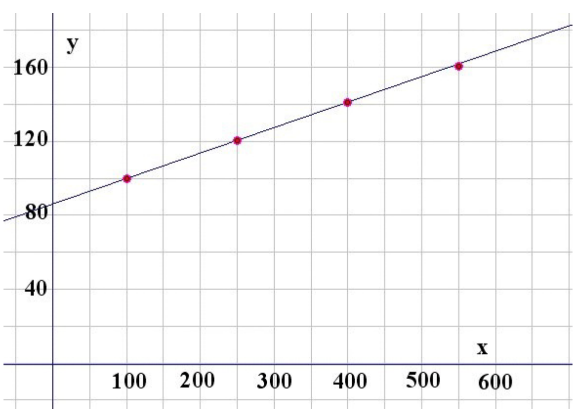
a. (–2, 1) (–5, 4) b. (2, –5) (–3, 4) c. (–3, 5) and (4, 1)

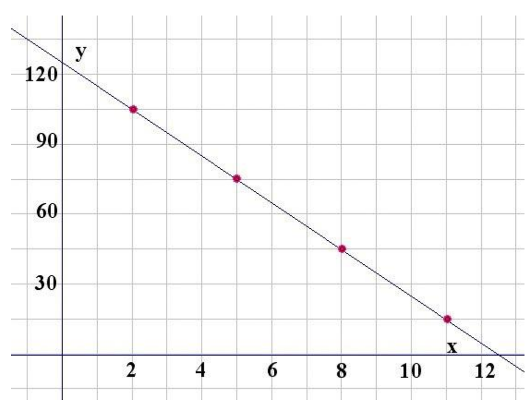






10. Find the slope of each line pictured below





11. Find the slope of the line that passes through each pair of points.

a. (1,2) and (7, 9) b. (-5, 3) and (-1,0) c. (5,-1) and (0, 3)

d. (6,2) and (6,-5)

e. (12,5) and (9,8)

f. (-3,-7) and (-8, -1)

g. (3,-5) and (0,0)

h. (2, ¾) and (4, ¼)

i. (½, 2/3) and (0, 1/3)

12. Use the slope formula to determine the slope of the line that passes through each set of points

a. (5, 7) and (2, 1)

b. (3, –5) and (–2, 4)

c. (–4, –5) and (6, –1)

d. (4, 6) and (4, –2)

e. (3, 4) and (–2, 8)

f. (–9.8, –3.1) and (–4.2, 7.3)

13. Given the slope of the line segment and the other end point, determine the unknown.

a. (0, 5), (*a*, 1), slope 2

b. (*b*, 0), (10, 9), slope

c. (17, 21), (*c*, 5), slope 32/8

d. (7, 12), (12, *d*), slope 6

e. (17, *e*), (32, 9), slope

f. (3, *f*), (7, 4), slope 75/12

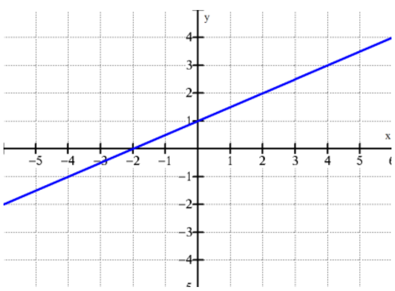
y-intercept

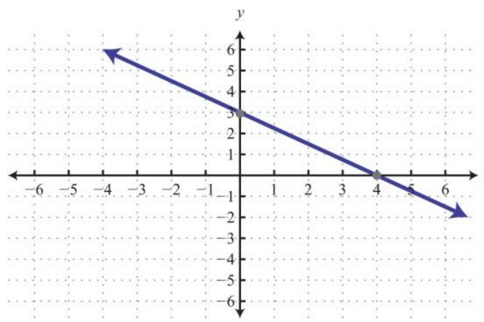
The **y-intercept** is the point where the **line crosses the y-axis**.

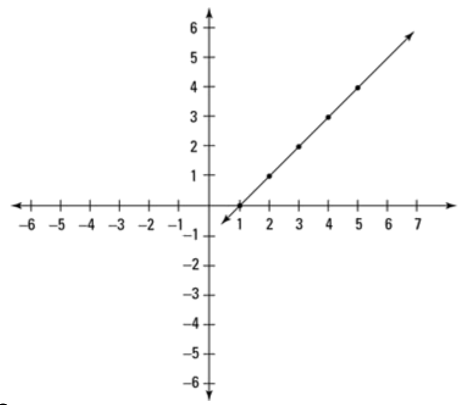
At the y-intercept, the **x-coordinate is always zero**. For example, (0, 12), (0, -4), (0, 13.56), and (0, -101) are all possible y-intercepts.

Another way of saying "y-intercept" is **"b"**

Ex 9. Identify the y-intercept on each graph.

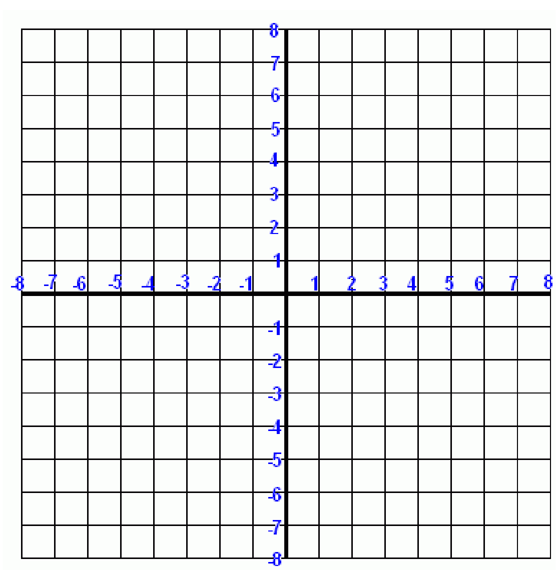




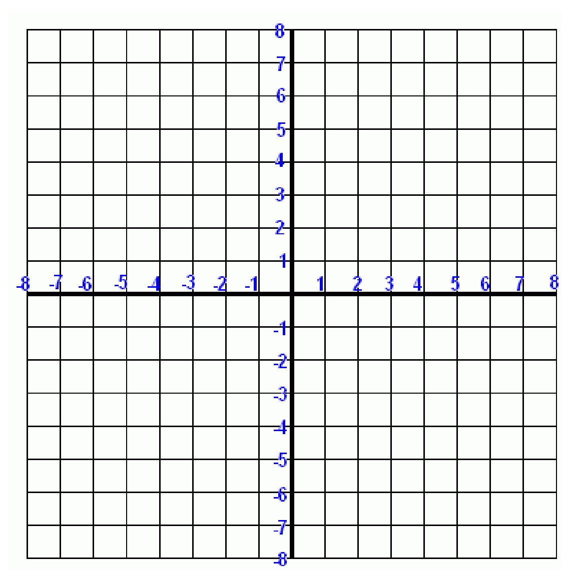


b= ______ b= ______ b= ______

Ex 10. Draw a line that has a y-intercept of –3, and a slope of $\frac{1}{2}$



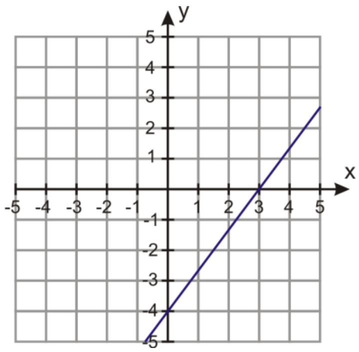
Ex 11. What is the y-intercept of a line that goes through the points (–5, –2) and (5, –5)?



Practice 3 – y-intercepts

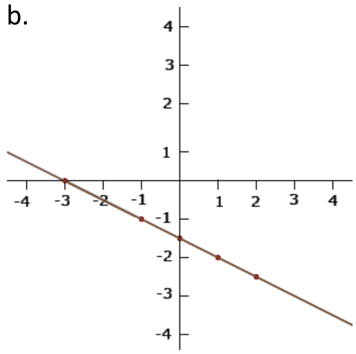
14. Identify the y-intercept for each line

a.



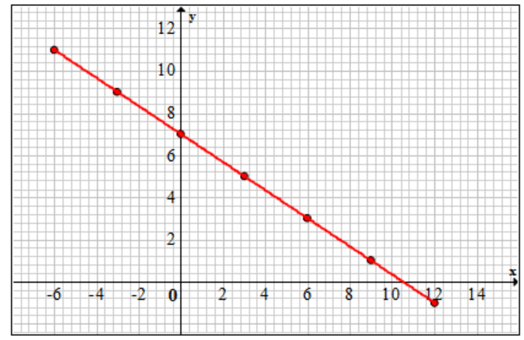
b = ______

b.



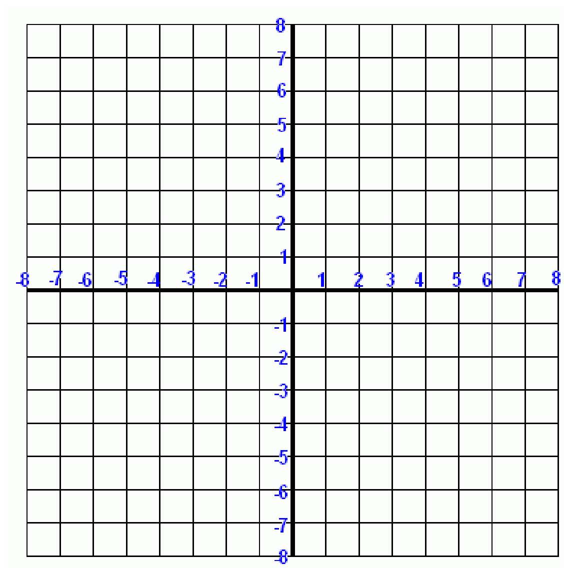
b = ______

c.

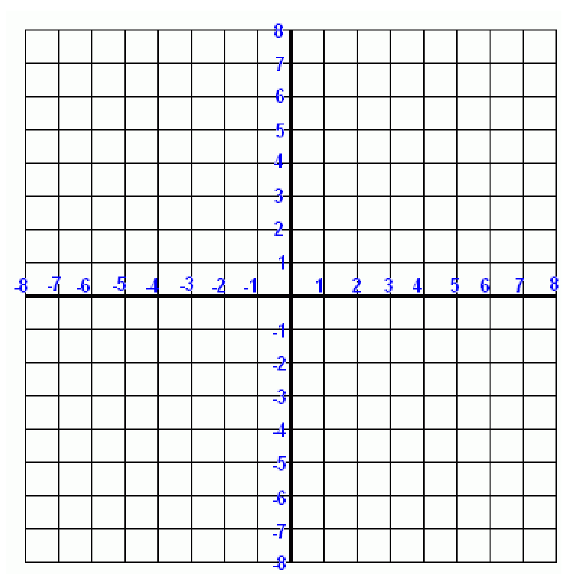


b = ______

15. Draw and label each line:
   a) a y-intercept of –2 and a slope of $\frac{1}{2}$
   b) slope of –5 and a y-intercept of 0
   c) b = –1 and m = $\frac{-3}{4}$
   d) The line intercepts the y-axis at 4, the rise is 2, and the run is 3

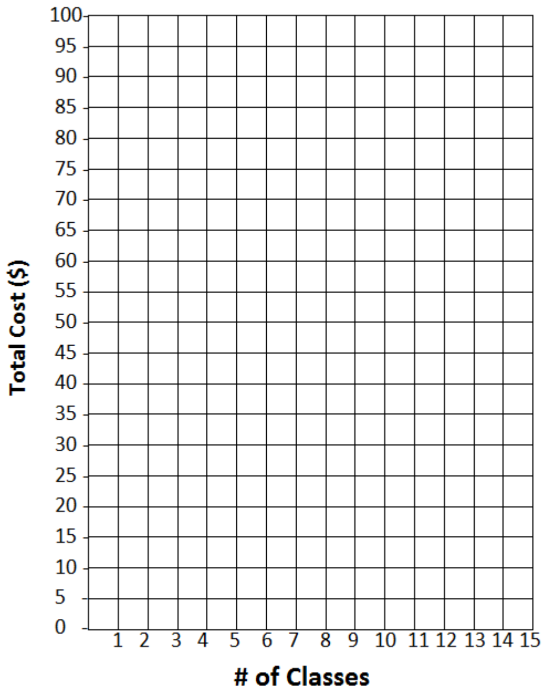
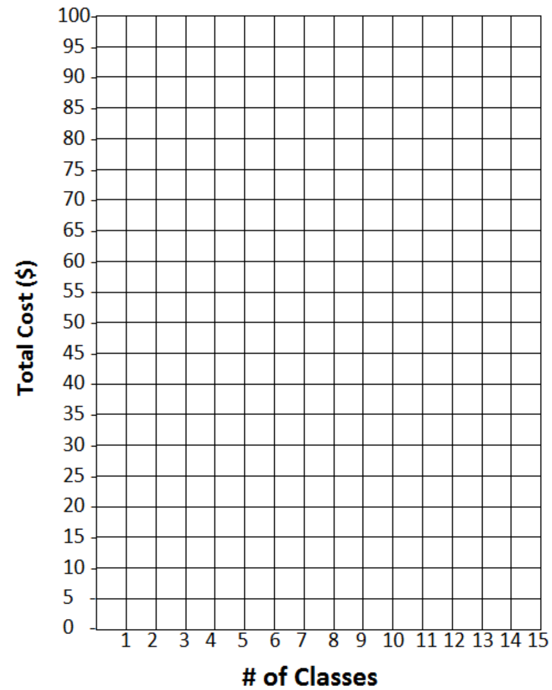


16. Draw each line and find the y-intercept
   a) Passes through (4,4) and (–4,0)
      b = ________
   b) Passes through (2,0) and (1, –3)
      b = ________
   c) Passes through (–4, –6) and (–8, –5)
      b = ________
   d) Passes through (–8, 3) and (4, –1.5)
      b = ________



Direct and Partial Variation

Ex 12. Isabella is looking to join a gym in order to take fitness classes. Here are the two fitness plans she is comparing:

<table>
<tr><td>In words:</td><td>Body by Ms. Will does not charge a membership fee, and charges $10 per fitness class.</td><td>Chahine’s Total Fitness charges a $40 membership fee, and $5 per fitness class.</td></tr>
<tr><td>In a graph:</td><td>
</td><td>
</td></tr>
<tr><td>In an equation:</td><td>Let y be the total cost
Let x be the # of classes
y = mx</td><td>Let y be the total cost
Let x be the # of classes
y = mx + b</td></tr>
<tr><td>In a table:</td><td><table><tr><th>x
(# of classes)</th><th>y
(total cost)</th></tr><tr><td>0</td><td></td></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td></td></tr><tr><td>5</td><td></td></tr><tr><td>6</td><td></td></tr><tr><td>7</td><td></td></tr><tr><td>8</td><td></td></tr><tr><td>9</td><td></td></tr><tr><td>10</td><td></td></tr></table></td><td><table><tr><th>x
(# of classes)</th><th>y
(total cost)</th></tr><tr><td>0</td><td></td></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td></td></tr><tr><td>5</td><td></td></tr><tr><td>6</td><td></td></tr><tr><td>7</td><td></td></tr><tr><td>8</td><td></td></tr><tr><td>9</td><td></td></tr><tr><td>10</td><td></td></tr></table></td></tr>
<tr><td></td><td>This is a DIRECT variation</td><td>This is a PARTIAL variation</td></tr>
</table>

Direct and Partial Variation

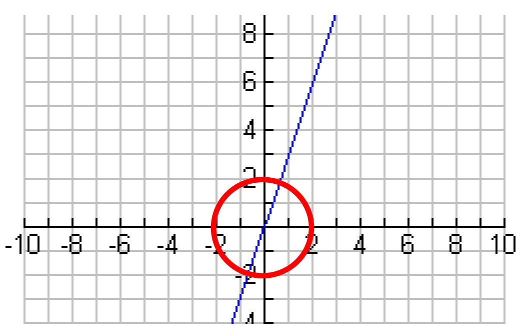
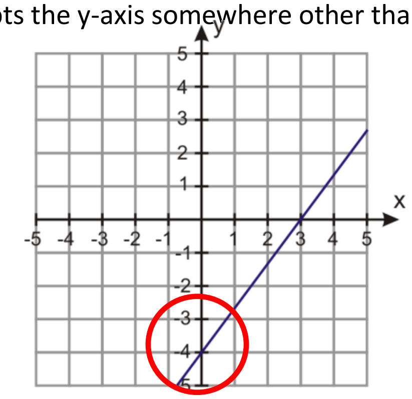
This is point-slope form of the equation of a line:

$$y = mx + b$$

m is called the variable rate, constant of variation, or slope

y and x are variables

b is called the constant, initial value, fixed cost, or y-intercept

<table>
<tr><th></th><th>Direct variation</th><th>Partial variation</th></tr>
<tr><td>In words…</td><td>Has a variable rate, but no fixed cost
- An electrician charges $30/hr for her services</td><td>Has a variable rate AND a fixed cost
- An electrician charges a fixed fee of $100 for a house call, and then $20/hr for her services</td></tr>
<tr><td>On a graph…</td><td>Passes through the origin

</td><td>Intercepts the y-axis somewhere other than at the origin

</td></tr>
<tr><td>In an equation…</td><td>Has the form y = mx
- y =5x</td><td>Has the form y = mx + b
- y = 5x + 7</td></tr>
<tr><td>In a table…</td><td>Will have the point (0,0)
<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>6</td></tr><tr><td>4</td><td>8</td></tr></table></td><td>Will not have the point (0,0)
<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>-1</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>3</td></tr><tr><td>3</td><td>5</td></tr><tr><td>4</td><td>7</td></tr></table></td></tr>
</table>

Homework: Pg 242-245 #1-6, 9, 12

Pg 250-253 C1, C3, #1-7, 9-11, 12

Practice 4 – Direct and Partial Variation

17. The depth of water in a tub varies directly as the length of time the taps are on. If the taps are left on for 4 minutes, the depth of water is 24cm.

a. By what number is the time multiplied to give the water depth?

b. Write the equation relating depth to time.

c. Find the depth of the water if the taps are left on for 5 minutes.

d. Find the length of time the taps were left on if the depth of the water is 42cm.

18. The following equation represents the cost, T, in dollars to print n pamphlets, in hundreds, $T = 200 + 25n$.

a. Construct a table of values for n and T.

b. In this printing job, what is the fixed cost and what is the variable cost?

c. What is the cost of printing 800 pamphlets? 2500 pamphlets?

d. How many pamphlets can be printed for $500?

19. At any given moment during daylight, the lengths of the shadows of objects vary directly as their heights. A tree 10m tall casts a shadow 16m long.

    a. By what number are the heights multiplied to give the shadow lengths?

    b. Write the equation relating shadow length to height.

    c. How long is the shadow if the tree is 32m tall?

    d. How tall is a tree that casts a shadow 24m long?

20. A car's rate of fuel consumption averages 8.0L/100km.If the fuel tank contains 60L of gasoline to begin with, make a table of values relating *n*, the number of litres of gasoline left in the tank, to *d*, the distance travelled in hundreds of kilometers.

    a. Write an equation relating *n* and *d*

    b. When the car has travelled 280km, about how much fuel is left in the tank?

First Differences

Calculating first differences is a way of using information from a chart to **determine if a relation is linear** or not, without actually graphing the data.

To find the first differences, we calculate the differences between consecutive y-values in a table of values that has evenly spaced x-values.

If the first differences are all the **same**, the relationship is **linear**

If the first differences are **not** all the same, the relationship is **non-linear**

Ex. 13

Since these are evenly spaced…

x	y
0	0
1	3
2	6
3	9
4	12

…we calculate the differences between these

What are the first differences?

x	y	First Differences
0	0	$3 - 0 = 3$
1	3	$6 - 3 = 3$
2	6	$9 - 6 = 3$
3	9	$12 - 9 = 3$
4	12	

Since the first differences for each pair of y-values are the same (all 3), then we can say that the relationship is linear

Ex. 14 Calculate the first differences for each table and determine whether the relationship is **linear**, **non-linear** or if it **cannot be determined**.

a.

x	y	First Differences
2	5	
4	10	
6	15	
8	20	
9	25	

This relationship is ________________________________

b.

x	y	First Differences
−2	2	
−1	4	
0	8	
1	16	
2	32	

This relationship is ________________________________

Homework: Pg 276-277 #2-4