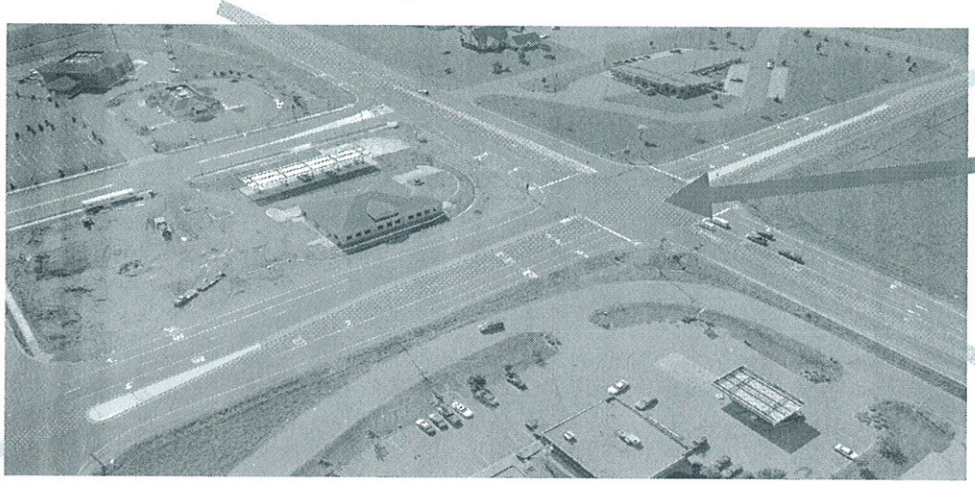
Solving a Linear System

If you have a **linear system**, or a system of linear equations, that means you have **two or more** linear equations. **Solving** a linear system means **finding the point where the lines intersect** (cross). The **solution** (or point of intersection) is a **point that lies on both lines**.



Solving systems of linear equations by graphing

Ex 6 Isabella is looking to join a gym in order to take fitness classes. Here are the two fitness plans she is comparing:

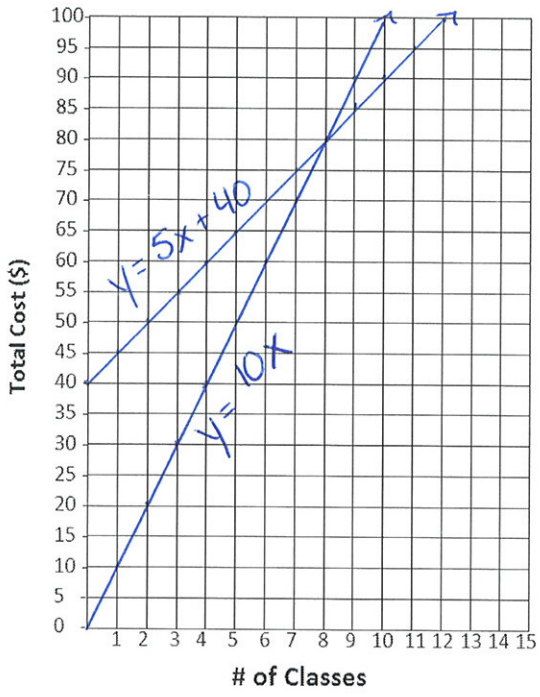
Body by Ms. Will does not charge a membership fee, and charges $10 per fitness class.

This can be represented by the equation $y = 10x$

Chahine's Total Fitness charges a $40 membership fee, and $5 per fitness class.

This can be represented by the equation $y = 5x + 40$

Let x represent number of classes
Let y represent total cost



a) Graph BOTH equations on the grid. Find the point of intersection. What does it mean? (8, 80)

8 classes will cost $80 at both gyms.

b) What recommendation would you make if Isabella thinks she will take about 6 classes?

Body by Ms. Will will be cheaper.

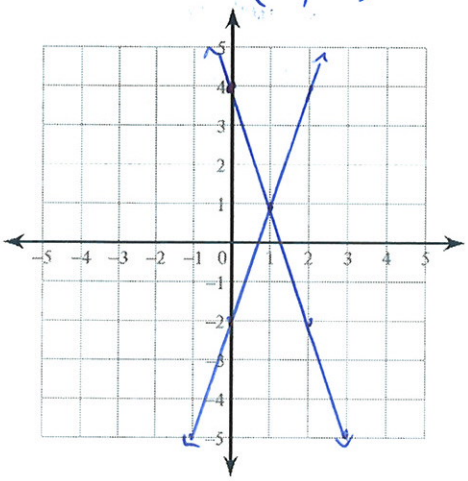
c) What recommendation would you make if Isabella thinks she will take 10 or more classes?

Chahine's Total Fitness will be cheaper.

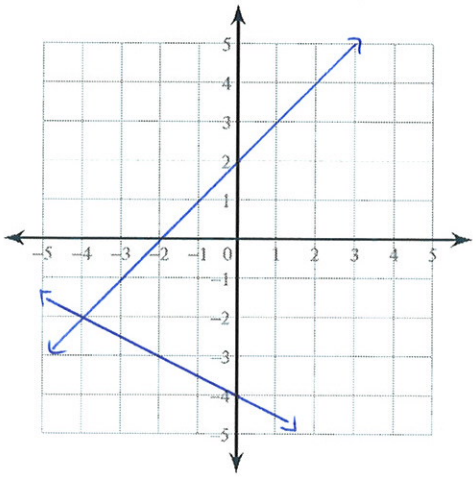
Practice 6 – Solving a System of Equations by Graphing

19. For each of the following systems of equations, find the solution by graphing.

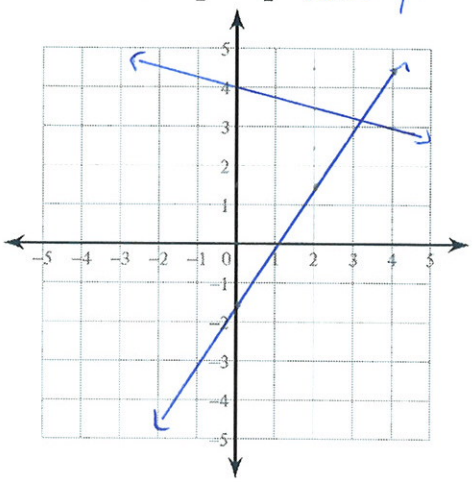
a. $y = -3x + 4$ and $y = 3x - 2$ (1, 1)



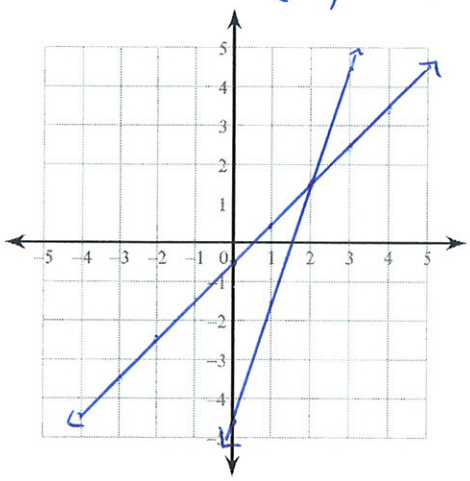
b. $y = x + 2$ and $y = \frac{-1}{2}x - 4$ (−4, −2)



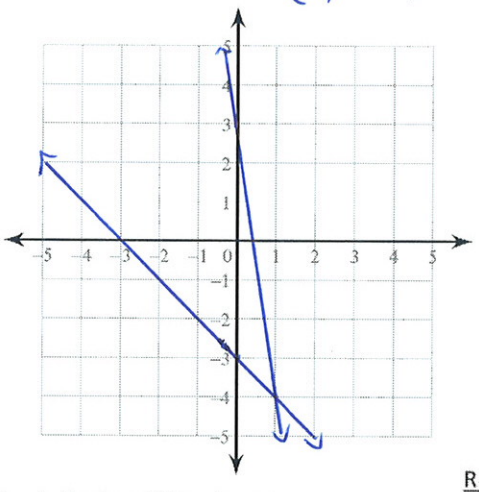
c. $y = -\frac{1}{4}x + 4$ and $y = \frac{3}{2}x - \frac{1}{2}$ (3.25, 3.25)



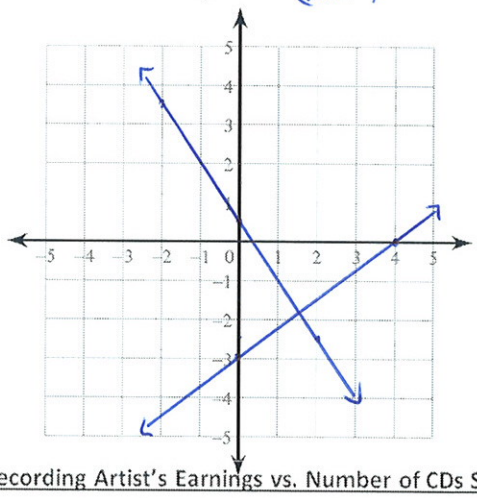
d. $y = x - \frac{1}{2}$ and $y = 3x - 4.5$ (2, 1.5)



e. $y = -x - 3$ and $y = 7x + 3$ (1, −4)



f. $y = -\frac{3}{2}x + \frac{1}{2}$ and $y = \frac{3}{4}x - 3$ (1.6, −1.7)



20. A recording artist is offered two deals for her CD release:

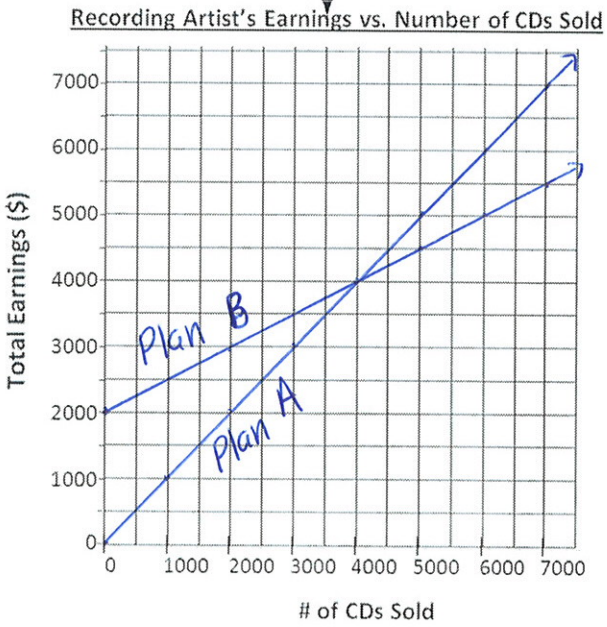
Plan A – Royalty only: \$1 per CD sold

Plan B – Partial royalty: \$2000 plus \$0.50 per CD sold.

a. Graph both linear relations on the same grid.

b. Find the solution of the linear system, and explain what it means.
(4000, 4000) 4000 CDs makes \$4000 for both earning plans.

c. What recommendations would you make for this artist? Give two different recommendations for different amounts of CDs sold. If the artist expects to sell fewer than 4000 CDs, they should go with Plan B. If they expect to sell more than 4000 CDs, they should go with Plan A.



Solving a Linear System

Solving systems of linear equations algebraically

Ex 7 Isabella is looking to join a gym in order to take fitness classes. Here are the two fitness plans she is comparing:

Body by Ms. Will does not charge a membership fee, and charges $10 per fitness class.

This can be represented by the equation $y = 10x$

Chahine's Total Fitness charges a $40 membership fee, and $5 per fitness class.

This can be represented by the equation $y = 5x + 40$

Let x represent number of classes
Let y represent total cost

Recall from when we graphed these lines that the solution to this linear system was **(8,80)**

We can get the same answer WITHOUT graphing.

How it works: We know that the solution to a system of equations gives us a point where the x and y are the same for BOTH equations. First, let's prove that.

Ex 7a) For both gyms, find the cost for 8 classes:

Body by Ms. Will:

$y = 10x$

$y = 10(8)$

$y = 80$

Therefore, the point $(8, 80)$ lies on this graph

Chahine's Total Fitness:

$y = 5x + 40$

$y = 5(8) + 40$

$y = 40 + 40$

$y = 80$

Therefore, the point $(8, 80)$ lies on this graph

Since $(8, 80)$ lies on both graphs, we know it is the point of intersection

Ex 7b) Now, let's find the point of intersection without graphing.

If the "y" for both gyms are the same, we can say: $y = y$

If $y = 10x$, AND $y = 5x + 40$, then we can say that: $10x = 5x + 40$

Now we have ONE equation with ONE type of variable, so we can solve for x.

$10x = 5x + 40$

$10x - 5x = 5x - 5x + 40$

$\frac{5x}{5} = \frac{40}{5}$

$\boxed{x = 8}$

$y = 10x$

$y = 10(8)$

$\boxed{y = 80}$

$\therefore (8, 80)$ is the solution to this system of linear equations

Now let's try another example:

Ex 8 Find the point of intersection between $y = 95.70x + 1200$ and $y = -63.50x - 140$

1	Make sure that both equations are in "y = mx + b" form	They are both in $y = mx + b$ form.
2	Set both equations equal to each other	$95.70x + 1200 = -63.50x - 140$
3	Solve for x	$95.70x + 63.50x = -140 - 1200$ $\frac{159.2x}{159.2} = \frac{-1340}{159.2}$ $x = -8.42$
4	Substitute x into either one of the two equations to find y	$y = 95.70(-8.42) + 1200$ $= -805.52 + 1200$ $= 394.49$
5	End with a point or a therefore statement	∴ The point of intersection is $(-8.42, 394.49)$
Optional	Check that this is the point of intersection by substituting your x and y values into EACH original equation, and performing a LS = RS check	LS = RS Check $y = 95.70x + 1200$ 394.49 | $95.70(-8.42)+1200$ $-805.794 + 1200$ 394.21 LS = RS ✓ LS = RS Check $y = -63.50x - 140$ 394.49 | $-63.50(-8.42)-140$ $534.67 - 140$ 394.67 LS = RS ✓ * numbers will not be exact due to rounding.

Practice 7 – Solving a System of Equations Algebraically

21. For each of the following systems of equations, find the point of intersection algebraically.

a. $y = 23.8x - 12$ and $y = -3x + 43$ $(2.05, 36.84)$

b. $y = \frac{1}{3}x - 7$ and $y = \frac{-2}{3}x + 13$ $(20, -\frac{1}{3})$

c. $y = 8x + \frac{3}{5}$ and $y = 9x - \frac{2}{5}$ $(1, 8.6)$

d. $y = \frac{1}{3}x + \frac{1}{2}$ and $y = \frac{1}{6}x - \frac{2}{3}$ $(-7, -\frac{11}{6})$

e. $y = 1900x - 320$ and $y = -1270x + 13000$ $(4.20, 7663.6)$

f. $y = -3.4x + 45.2$ and $y = 294.1x - 184.2$ $(0.77, 42.58)$