Slope

Slope of an object

When we find the **slope** of an object, we are finding how **steep** it is.

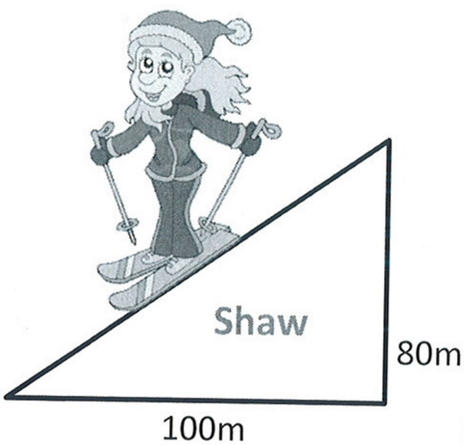
To find slope, we find the height of the object, and divide it by the length. Here it is in an equation:

$$\boxed{\text{Slope} = \frac{\text{rise}}{\text{run}}}$$

rise ← Height (vertical)

run ← Length (horizontal)

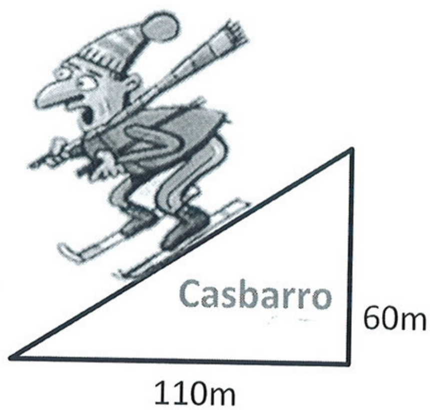
Ex 1. Mme. Shaw, Mr. Casbarro, and Mr. Ma all go skiing. Each takes a different run, and when they get back to the chalet, they argue over who is the biggest daredevil. Each insists that their ski run was the steepest. They know the heights and lengths of the ski runs. **Find who skied the steepest run.**



$\text{Slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{80}{100}$

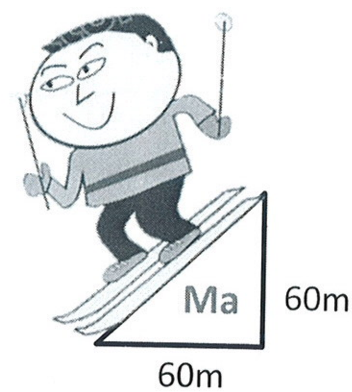
$= 0.8$



$\text{Slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{60}{110}$

$= 0.55$



$\text{Slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{60}{60}$

$= 1$

∴ Mr. Ma skiied the steepest run.

Ex 2. Building regulations state that wheelchair ramps cannot have a slope greater than $\frac{1}{12}$. If the height of a wheelchair ramp is 1.5, what is the ~~maximum~~ minimum length it can have?

$\text{slope} = \frac{\text{rise}}{\text{run}}$

$\frac{1}{12} = \frac{1.5}{x}$

$x = 1.5(12)$

$x = 18$

∴ The maximum length it can have is 18 m. This would give it a slope of $\frac{1}{12}$.

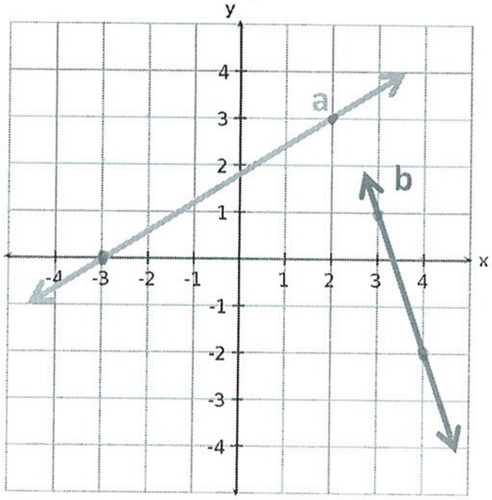
Slope of a Line

Slope is sometimes represented by the letter "m"

$$\textbf{Slope} = \text{m} = \frac{\textbf{rise}}{\textbf{run}}$$

...from a graph

Ex 3. Find the slopes of each line.



Slope of a

$m = \frac{3}{5}$

or

$= 0.6$

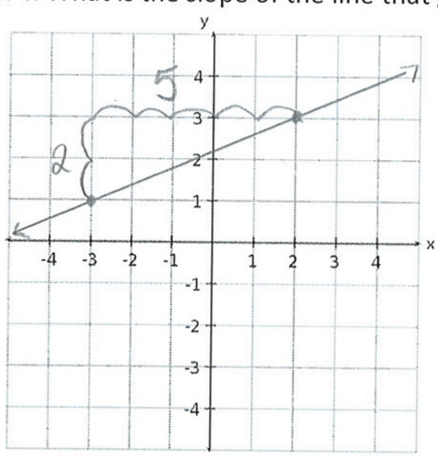
Slope of b

$m = \frac{3}{-1}$

$= -3$

...from two points

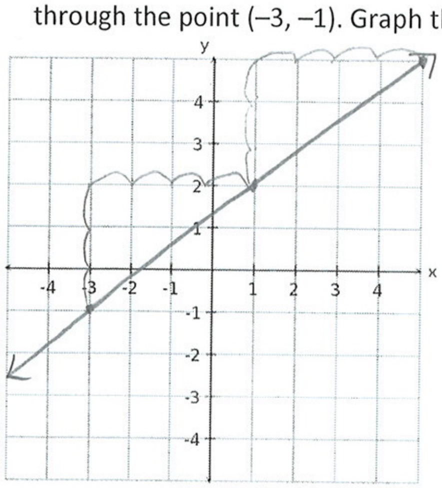
Ex 4. What is the slope of the line that goes through points **C(–3, 1)** and **D(2,3)**?



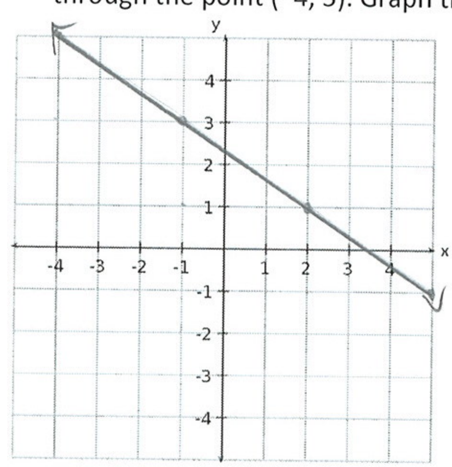
$m = \frac{2}{5}$

...graphing from a slope and a point

Ex 5. A line has a slope of $\frac{3}{4}$ and passes through the point (–3, –1). Graph this line.



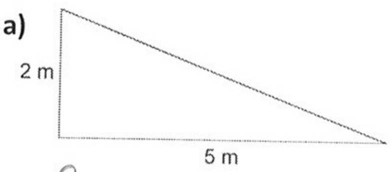
Ex 6. A line has a slope of $\frac{-2}{3}$ and passes through the point (–4, 5). Graph this line.



Practice 1 – Slope of Objects

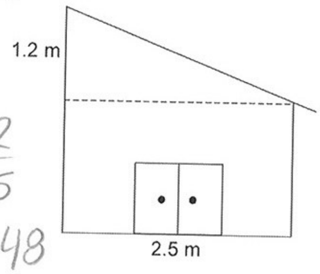
1. Find the slope of each object/line segment.

a)



$\frac{2}{5} = 0.4$

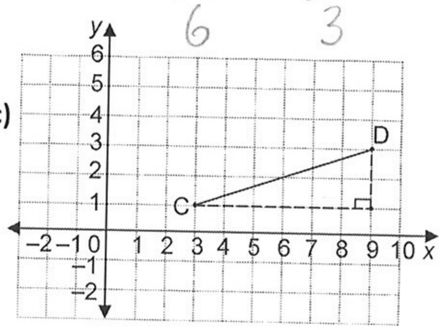
b)



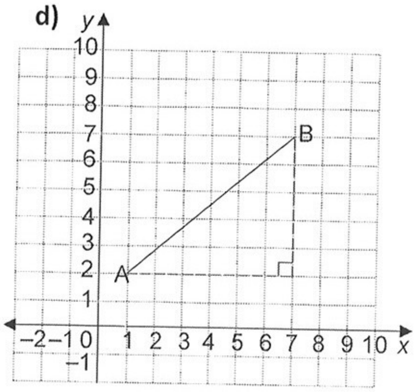
$\frac{1.2}{2.5} = 0.48$

c)

$\frac{-2}{6} = \frac{-1}{3}$

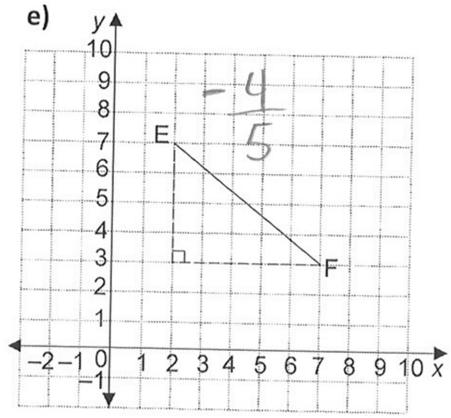


d)



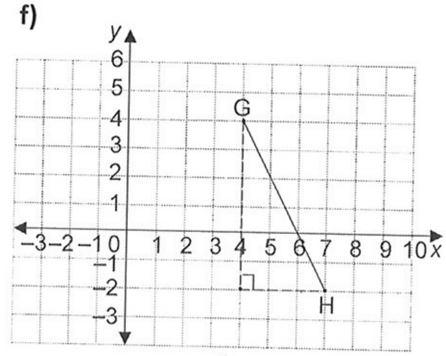
$\frac{5}{6}$

e)



$\frac{-4}{5}$

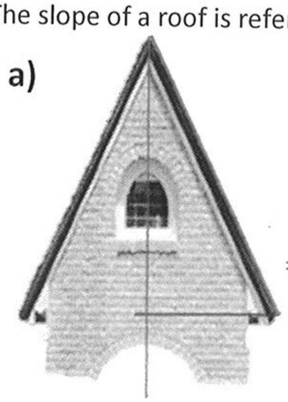
f)



$-\frac{6}{3} = -2$

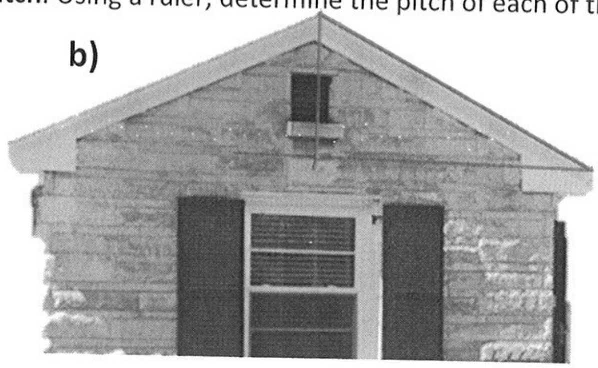
2. The slope of a roof is referred to as its **pitch**. Using a ruler, determine the pitch of each of these roofs.

a)



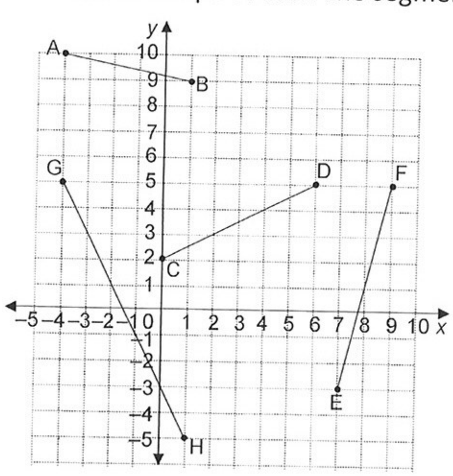
$\frac{3.7}{1.8} = 2.06$

b)



$\frac{2}{3.7} = 0.54$

3. Find the slope of each line segment.



$\overline{AB} = \frac{-1}{5}$

$\overline{CD} = \frac{3}{6} = \frac{1}{2}$

$\overline{EF} = \frac{8}{2} = 4$

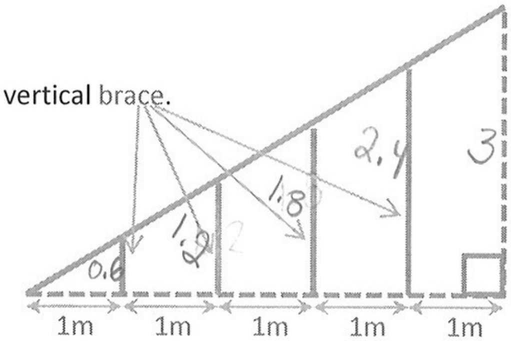
$\overline{GH} = \frac{-10}{5} = -2$

4. A ramp needs to have a slope of $\frac{3}{5}$. determine the length of each vertical brace.

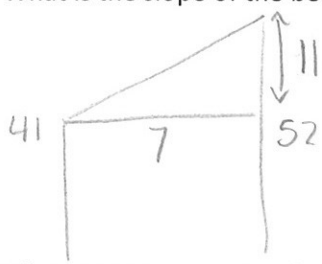
$\frac{3}{5} = \frac{x}{1}$

$\frac{5x}{5} = \frac{3}{5}$

$x = \frac{3}{5}$

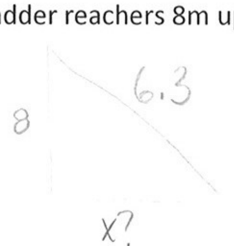


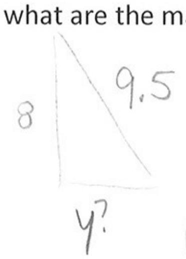
5. A steel beam goes between the tops of two buildings that are 7m apart. One building is 41m tall. The other is 52m tall. What is the slope of the beam?



$\frac{11}{7} = 1.57$

6. For safety reasons, an extension ladder should have a slope of between 6.3 and 9.5 when it is placed against a wall. If a ladder reachers 8m up a wall, what are the maximum and minimum distances from the foot of the ladder to the wall?





$\frac{8}{x} = 6.3$

$8 = 6.3x$

$1.27 = x$

Max

$\frac{8}{y} = 9.5$

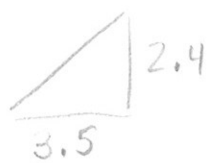
$8 = 9.5y$

$0.84 = y$

Min

7. For safety, the slope of a staircase must be greater than 0.58 and less than 0.70.
A staircase has a vertical rise of 2.4 m over a horizontal run of 3.5 m.

a) Find the slope of the staircase.

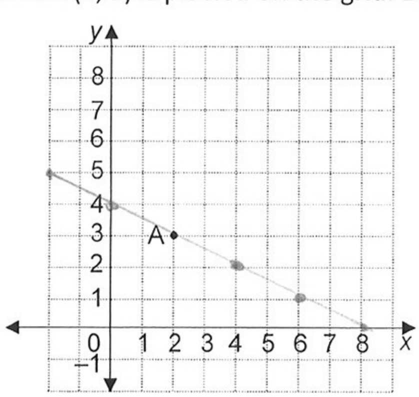


$\frac{2.4}{3.5} = 0.6857$

b) Is the staircase safe?

Yes. 0.69 is between 0.58 and 0.70.

8. Point A (2, 3) is plotted on the grid. Draw a line segment AB with slope $-\frac{1}{2}$. What are possible coordinates of B?



B could be: (−2,5)

(0,4)

(4,2)

(6,1)

(8,0)

etc

Slope as a rate of change

Slope represents the **steepness** of a line on a graph, but it is also a **rate of change**.

A **rate of change** is a change in one quantity relative to the change in another quantity.

When we represent a slope as a rate of change, we need to include units, such as **kilometres per hour**.

Here is the new way we can represent slope:

$$\textbf{Slope} = \frac{\Delta y}{\Delta x} = \frac{\textit{change in } y}{\textit{change in } x}$$

To calculate how a value changes, we take the end value and from it, we subtract the beginning value. For example:

Jenna's weight increased from 107 lbs to 113 lbs. By how much did her weight **change**?

change = end value – beginning value
= 113 – 107
= 6

∴ her weight changed by 6 lbs

So, if we had two "y" values, we would represent change in these values as $y_2 - y_1$

And if we had two "x" values, we would represent change in these values as $x_2 - x_1$

Where (x_1, y_1) and (x_2, y_2) are two points on the graph/in the table.

This means, that slope can be represented as follows:

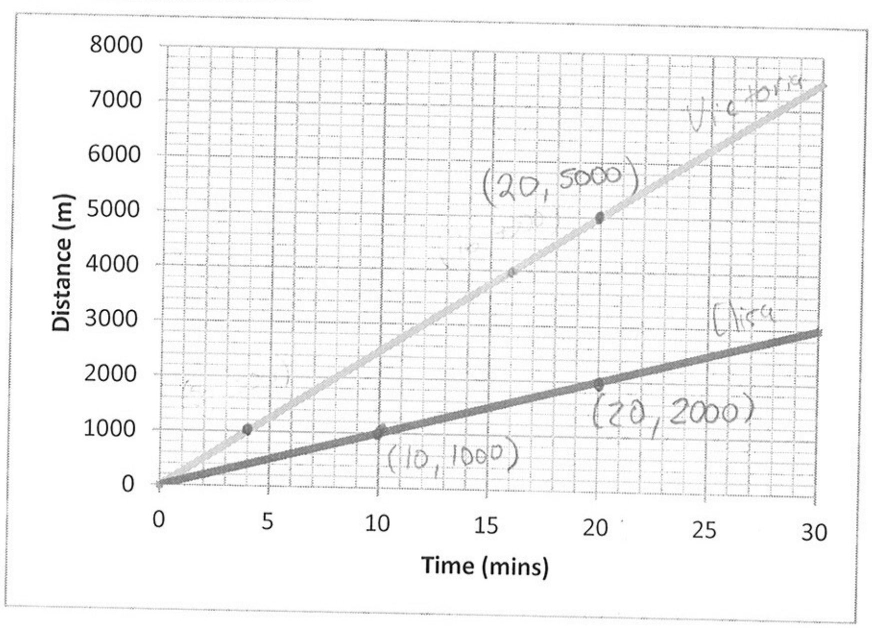
(x_2, y_2) is one point on the line

$$\textbf{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

(x_1, y_1) is another point on the line

Slope as a rate of change from a graph

Ex 7. Victoria and Elisa both run every morning. The following graph shows the distance they travelled over time.



a) Find the speed that each girl travelled (speed is calculated as $\frac{\text{distance}}{\text{time}}$).

	Victoria	Elisa
Choose two points on the graph	(x, y) (0, 0) ÷ (x, y) (20, 5000)	(x, y) (10, 1000) ÷ (x, y) (20, 2000)
Plug them into the formula	$\frac{y_2 - y_1}{x_2 - x_1}$ $\frac{5000 - 0}{20 - 0}$	$\frac{2000 - 1000}{20 - 10}$
Simplify the top and the bottom	$\frac{5000}{20}$	$\frac{1000}{10}$
Divide, or leave as a fraction if not evenly divisible	$\frac{500}{2}$ = 250 m/s	100 m/s

b) Who ran faster? What was her speed?

Victoria. 250 m/s

c) What does a faster speed look like on a graph?

Steeper slope

Slope as a rate of change from a table of values

Ex 8. This table of values shows the volume of gasoline remaining in a car's tank. It forms a linear relationship.

Distance (km)	0	100	200	300	400	500
Volume (L)	65	53	41	29	17	5

a) Calculate the slope of the line that would result from these data points.

Choose two points on the graph	(0, 65) (100, 53)
Plug them into the formula	$\frac{53 - 65}{100 - 0}$ =
Simplify the top and the bottom	$\frac{-12}{100}$
Divide, or leave as a fraction if not evenly divisible	$-\frac{3}{25}$ or = -0.12