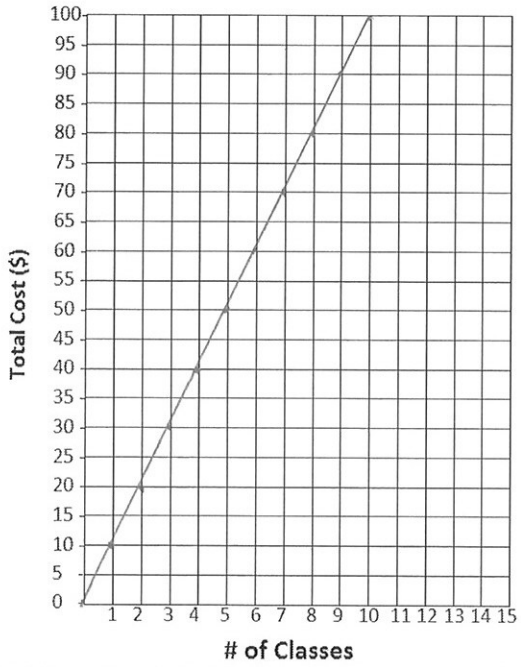
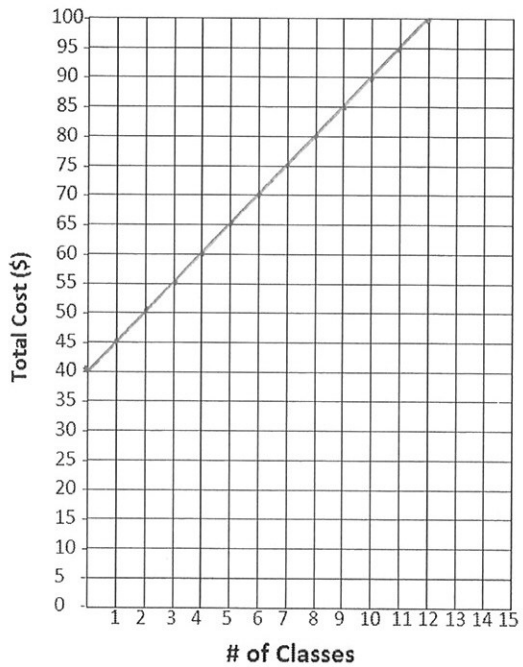
Direct and Partial Variation

Ex 12. Isabella is looking to join a gym in order to take fitness classes. Here are the two fitness plans she is comparing:

In words:	**Body by Ms. Will** does not charge a membership fee, and charges $10 per fitness class.	**Chahine's Total Fitness** charges a $40 membership fee, and $5 per fitness class.
In a graph:		
In an equation:	**Let y be the total cost** **Let x be the # of classes** **$y = mx$** $y = 10x$	**Let y be the total cost** **Let x be the # of classes** **$y = mx + b$** $y = 5x + 40$
In a table:	x (# of classes) / y (total cost) 0 / 0 1 / 10 2 / 20 3 / 30 4 / 40 5 / 50 6 / 60 7 / 70 8 / 80 9 / 90 10 / 100	x (# of classes) / y (total cost) 0 / 40 1 / 45 2 / 50 3 / 55 4 / 60 5 / 65 6 / 70 7 / 75 8 / 80 9 / 85 10 / 90
	This is a **DIRECT** variation	This is a **PARTIAL** variation

Direct and Partial Variation

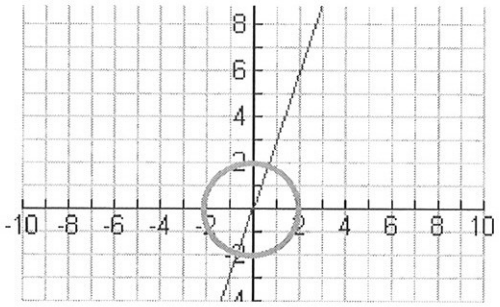
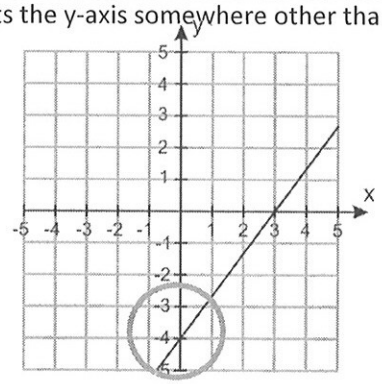
This is point-slope form of the equation of a line:

$$y = mx + b$$

m is called the variable rate, constant of variation, or slope

y and x are variables

b is called the constant, initial value, fixed cost, or y-intercept

	Direct variation	Partial variation
In words...	Has a variable rate, but no fixed cost - *An electrician charges $30/hr for her services*	Has a variable rate AND a fixed cost - *An electrician charges a fixed fee of $100 for a house call, and then $20/hr for her services*
On a graph...	Passes through the origin 	Intercepts the y-axis somewhere other than at the origin 
In an equation...	Has the form y = mx - *y =5x*	Has the form y = mx + b - *y = 5x + 7*
In a table...	Will have the point (0,0) x | y 0 | 0 1 | 2 2 | 4 3 | 6 4 | 8	Will not have the point (0,0) x | y 0 | -1 1 | 1 2 | 3 3 | 5 4 | 7

Homework: Pg 242-245 #1-6, 9, 12
Pg 250-253 C1, C3, #1-7, 9-11, 12

Practice 4 – Direct and Partial Variation

17. The depth of water in a tub varies directly as the length of time the taps are on. If the taps are left on for 4 minutes, the depth of water is 24cm.

a. By what number is the time multiplied to give the water depth?

$y = mx$

$24 = m(4)$

$\frac{24}{4} = \frac{4m}{4}$

$\boxed{6 = m}$

∴ It is multiplied by 6.

b. Write the equation relating depth to time.

$\boxed{y = 6x}$

c. Find the depth of the water if the taps are left on for 5 minutes.

$y = 6(5)$

$\boxed{y = 30}$

∴ The depth is 30cm.

d. Find the length of time the taps were left on if the depth of the water is 42cm.

$y = 6x$

$\frac{42}{6} = \frac{6x}{6}$

$\boxed{7 = x}$

∴ They were on for 7 minutes.

18. The following equation represents the cost, *T*, in dollars to print *n* pamphlets, in hundreds, $T = 200 + 25n$.

a. Construct a table of values for *n* and *T*.

n	T
0	200
1	225
2	250
3	275
4	300
5	325

b. In this printing job, what is the fixed cost and what is the variable cost?

fixed cost → 200

variable cost → 25

c. What is the cost of printing 800 pamphlets? 2500 pamphlets?

$T = 200 + 25(8)$

$= 400$

∴ It costs $400 for 800.

$T = 200 + 25(25)$

$= 825$

∴ It costs $825 for 2500.

d. How many pamphlets can be printed for $500?

$T = 200 + 25n$

$500 = 200 + 25n$

$\frac{300}{25} = \frac{25n}{25}$

$12 = n$

∴ 1200 can be printed

19. At any given moment during daylight, the lengths of the shadows of objects vary directly as their heights. A tree 10m tall casts a shadow 16m long.

a. By what number are the heights multiplied to give the shadow lengths?

Let y be shadow length, let x be height

$y = mx$

$\frac{16}{10} = \frac{m(10)}{10}$ → $1.6 = m$

b. Write the equation relating shadow length to height.

$y = 1.6x$

c. How long is the shadow if the tree is 32m tall?

$y = 1.6(32)$

$y = 51.2$

∴ The shadow is 51.2 m long.

d. How tall is a tree that casts a shadow 24m long?

$\frac{24}{1.6} = \frac{1.6x}{1.6}$

$15 = x$

∴ The tree is 15m tall

20. A car's rate of fuel consumption averages 8.0L/100km.If the fuel tank contains 60L of gasoline to begin with, make a table of values relating *n*, the number of litres of gasoline left in the tank, to *d*, the distance travelled in hundreds of kilometers.

a. Write an equation relating *n* and *d*

$n = -8d + 60$

d	n
0	60
1	52
2	44
3	36
4	28

b. When the car has travelled 280km, about how much fuel is left in the tank?

$n = -8(2.8) + 60$

$n = -22.4 + 60$

$n = 37.6$

∴ About 37.6 L are left in the tank

First Differences

Calculating first differences is a way of using information from a chart to **determine if a relation is linear** or not, without actually graphing the data.

To find the first differences, we calculate the differences between consecutive y-values in a table of values that has evenly spaced x-values.

If the first differences are all the **same**, the relationship is **linear**

If the first differences are **not** all the same, the relationship is **non-linear**

Ex. 13

Since these are evenly spaced...

x	y
0	0
1	3
2	6
3	9
4	12

...we calculate the differences between these

What are the first differences?

x	y	First Differences
0	0	
1	3	3 – 0 = 3
2	6	6 – 3 = 3
3	9	9 – 6 = 3
4	12	12 – 9 = 3

Since the first differences for each pair of y-values are the same (all 3), then we can say that the relationship is linear

Ex. 14 Calculate the first differences for each table and determine whether the relationship is **linear**, **non-linear** or if it **cannot be determined**.

a.

x	y	First Differences
2	5	
4	10	10 – 5 = 5
6	15	15 – 10 = 5
8	20	20 – 15 = 5
9	25	25 – 20 = 5

This relationship is non-linear

b.

x	y	First Differences
–2	2	
–1	4	4 – 2 = 2
0	8	8 – 4 = 4
1	16	16 – 8 = 8
2	32	32 – 16 = 16

This relationship is non-linear

Homework: Pg 276-277 #2-4